SU(5) GUTs from compact Type IIB orientifolds

based on

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Motivation

GUT model building classic topos in string phenomenology since 1985

new impulses from recent GUT model building advances in F-theory

Donagi/Wijnholt and Beasley/Heckman/Vafa 2008

F-theory:

- IIB compactification with D-branes on fully backreacted manifold
- sensitive to non-perturbative effects beyond Type IIB

work on F-theory GUTs so far: properties of local quivers

except: Tatar et al.'08; Andreas,Curio; Donagi,Wijnholt; Marsano et al. '09

Global consistency conditions are at the heart of string theory:
distinguish string landscape from swampland

at present consistency conditions better understood in Type IIB language (esp. gauge flux)

most promising avenue for unification from F-theory is via SU(5) GUT
$\Rightarrow$ This is (and has been for a while) amenable to Type II methods

Motivation

This talk:
Systematic analysis of SU(5) GUTs in IIB vacua

Aim: implementation of GUT quivers into **actual string vacua as opposed to local quivers**

- Explicit **Type IIB vacua serve as starting point for F-theory** models upon uplifting
  Do useful geometric properties of Type IIB Calabi-Yau 3-fold survive?
- Ultimate goal:
  model building in combination with moduli stablisation
  $\leftrightarrow$ required for satisfactory discussion of SUSY breaking, predictions...
  Type IIB orientifolds on genuine (conformal) Calabi-Yau promising

Outline

1) Motivation

2) Background on Type IIB orientifolds with D3/D7-branes

- gauge flux
- global consistency conditions
- massless matter

3) SU(5) GUT model building in Type IIB orientifolds

- GUT breaking
- non-perturbative Yukawa couplings

4) Explicit construction of semi-realistic SU(5) vacua

5) Conclusions

Type IIB Orientifolds

- Compactification of Type IIB theory on Calabi-Yau X
- orientifold: divide by $\Omega(-1)^{F_L}\sigma$, σ : holomorphic involution of X

$$\sigma^* J = J, \quad \sigma^*\Omega = -\Omega$$

split into even/odd cycles : Grimm, Louis '05

$h^{1,1}_+$ Kähler moduli $T_I = \int_{\Gamma_I^+} e^{-\phi} J \wedge J + iC_4$,

$h^{1,1}_-$ B-field moduli $G_i = \int_{\gamma_i^-} -B + iC_2$

discrete B-field parameter $\frac{1}{2\pi}\int_{\gamma_i^+} B = 0, \frac{1}{2}$

$\Rightarrow$ fix-point set O3/O7-planes $\leftrightarrow$ spacetime-filling D3/D7-branes

- D3-brane: point on internal X
- D7-brane: wraps holomorphic 4-cycle (divisor) D_a

upstairs geometry: D_a + image D'_a

1) D_a not invariant: $D_a \to D'_a \Rightarrow$ gauge group $U(N_a)$

2) D_a invariant: $2N_a \Rightarrow SO(2N_a)/Sp(2N_a)$

Gauge Flux

Gauge flux on D-brane: $\mathcal{F}_a = F_a + \iota^* B$

focus on $\langle \mathcal{F}_a \rangle \neq 0$ for abelian subgroups $\Leftrightarrow$ line bundles L_a

- typical embedding: diagonal $U(1) \subset U(N) \rightarrow SU(N) \times U(1)$, $U(1)$ massive by Green-Schwarz mechanism
- can also switch on $U(1) \subset SU(N)$

gauge flux on divisor $D \Leftrightarrow \langle F \rangle \in H^2(D) \Leftrightarrow$ 2-cycle $\in H_2(D)$

2 types of non-trivial 2-cycles on D: Lerche, Mayr, Warner'01/02;

non-trivial also on X vs. boundaries of 3-chains on X Jockers,Louis'05

$$\text{splitting} \quad L_a = \underbrace{\iota^* \mathbb{L}_a}_{\text{pullback from X}} \otimes \underbrace{R_a}_{\text{trivial on X}}$$

flux R_a does

- not affect chiral spectrum and
- not participate in GS mechanism Buican et al.'06

Global consistency conditions (I)

1) Freed-Witten quantisation condition on line bundles

path-integral of open string worldsheet with boundary on single U(1) brane D must be well-defined   Freed, Witten '99

Result: shift in Dirac quantisation condition

$c_1(L) - \iota^* B + \frac{1}{2} c_1(K_D) \in H^2(D, \mathbb{Z})$

$\Rightarrow$ L half-integer quantised

- for discrete B-field $\int_{\gamma_+} B = \frac{1}{2}$
- for divisor D not spin, i.e. $c_1(K_D) \in H^2(D, (2\mathbb{Z}+1)/2)$

choice of B-field determines quantisation on several divisors at once!

Generalisation to more general embedding: Blumenhagen,Braun,Grimm,T.W.'08

$T_0 \, (c_1(L_a^{(0)}) - \iota^* B) + \sum_i T_i \, c_1(L_a^{(i)}) + \frac{1}{2} T_0 \, c_1(K_{D_a}) \in H^2(D_a, \mathbb{Z})_{N_a \times N_a}$

$\Rightarrow$ suitably fractional line bundles are allowed!

Global consistency conditions (II)

2) Tadpole cancellation condition from CS action of D-brane and O-plane

- cancellation of D7-charge and induced D5-charge

see also: Collinucci,Esole,Denef; Plauschinn'08

- D3 :

$$N_{\rm D3} + \frac{N_{\rm flux}}{2} - \sum_a \int_{D_a} \frac{{\rm tr}\, \mathcal{F}_a^2}{8\pi^2} = \underbrace{\frac{N_{\rm O3}}{4} + \frac{\chi(D_{\rm O7})}{12} + \sum_a N_a \frac{\chi_o(D_a)}{24}}_{\chi(CY_4)/12 \,{\rm in\, F-theory}}$$

constraint by integrality of $N_{D3} \in \mathbb{Z}_0^+ \leftrightarrow$ Freed-Witten quantisation!

models with non-spin divisors tricky!

3) D-term supersymmetry

Fayet-Iliopoulos D-term: $\xi \;\sim\; \int_D \iota^* J \wedge c_1(L)$ Marino et al.'99

$\xi_a = 0 \rightarrow -\frac{N_a}{2} \int_{D_a} c_1^2(L_a) \geq 0$ Blumenhagen,Braun,Grimm,T.W.'08

$\Rightarrow$ SUSY bundles always contribute positively to D3-tadpole

$\leftrightarrow$ danger of overshooting

Massless Matter

adjoint chiral fields: $h^{(0,2)}(D)$ deformation and $h^{(0,1)}(D)$ Wilson moduli

charged chiral matter at intersection of D-branes along divisors D_a and D_b

open strings in

- $a \longrightarrow b$ sector: bifundamental matter $(\overline{\square}_{a(-1)}, \square_{b(1)})$
- $a' \longrightarrow a$ sector: (anti-)symmetric matter $\begin{smallmatrix}\square\\\square\end{smallmatrix}_{(2)}$ / $\square\square_{(2)}$

  - $D_a = D_b \longrightarrow$ matter localised on whole divisor D_a
    $\exists$ generically vector-like pairs
  - $D_a \neq D_b \longrightarrow$ matter localised on curve $C_{ab} = D_a \cap D_b$
    generically no vector-like pairs

either case: index $I_{ab} = -\int_X [D_a] \wedge [D_b] \wedge (\, c_1(L_a) - c_1(L_b)\,)$

relative flux only affects vector-like spectrum

SU(5) GUTs

starting point: $U(5)_a \times U(1)_b$ theory $\qquad$ $U(1)_{a,b}$ massive

GUT brane: (D_a, L_a), $\qquad$ U(1) brane (D_b, L_b)

sector	reps.	particle	sector	reps.	particle
(a', a)	$\mathbf{10}_{(2,0)}$	(Q_L, u^c_R, e^c_R)	(b', b)	$\mathbf{1}_{(0,2)}$	N^c_R
(a, b')	$\bar{\mathbf{5}}_{(-1,-1)}$	(d^c_R, L)	(a, b)	$\mathbf{5}^H_{(1,-1)} + \bar{\mathbf{5}}^H_{(-1,1)}$	$(T^u, H^u) + (T^d, H^d)$

matter localised on intersection loci

$\mathbf{10}_{(2,0)} \iff H^*(D_a \cap D_{a'}, L_a^2)$ $\qquad$ no vectorlike exotics:

$\bar{5}^m_{(-1,-1)} \iff H^*(D_a \cap D_{b'}, L_a^{-1} \otimes L_{b'})$ $\qquad$ $D_a \neq D_{a'}$

Two major challenges:

- complete description of GUT breaking:
  same solution exists in IIB/F-theory
  cannot be treated locally - depends on global data!

- Yukawa couplings:
  distinct approaches in IIB/ F-theory

SU(5) GUT breaking

Idea: **Embed** $U(1)_Y \subset U(5)$ [Beasley,Heckman,Vafa; Donagi,Wijnholt '08]

Approach very sensitive to global consistency:

1) $U(1)_Y$ massless iff $\mathcal{L}_Y$ in relative cohomology

rigid GUT divisor must possess 2-cycles trivial on ambient space

global feature - depends on compactification details, not on local data!

2) Freed-Witten quantisation: Blumenhagen,Braun,Grimm,T.W. '08

$\mathcal{L}_a \leftrightarrow T_a, \quad \mathcal{L}_Y \leftrightarrow \frac{2}{5}T_a + \frac{1}{5}T_Y \qquad T_a = 1_{5\times 5}, T_Y = \mathrm{diag}(-2,-2,-2,3,3)$

GUT breaking: $U(5)_a \rightarrow SU(3) \times SU(2) \times U(1)_Y \times U(1)_a$

From general quantisation condition:

$c_1(\mathcal{L}_a) - \iota^* B + \frac{1}{2} K_{D_a} \in \mathbb{Z}, \quad c_1(\mathcal{L}_a) + c_1(\mathcal{L}_Y) - \iota^* B + \frac{1}{2} K_{D_a} \in \mathbb{Z}$

- for non-spin divisor D_a and $B = 0$:
  $\mathcal{L}_a \neq \mathcal{O}$ such that $\mathcal{L}_a$ half-integer quantised
- $\mathcal{L}_Y$ integer quantised

Top Yukawas

Yukawa couplings from triple intersection of 3 matter curves

Problem: $\mathbf{10}^{(2,0)}\,\mathbf{10}^{(2,0)}\,\mathbf{5_H}^{(1,-1)}$ forbidden perturbatively in Type IIB

- **Solution: Stringy D-brane instantons** Blumenhagen,Cvetič,Weigand; Ibáñez,Uranga; Florea,Kachru,McGreevy,Saulina '06

  Euclidean D3-brane along divisor Ξ with $\Xi \cap D_{a,b} \neq 0$
  $\rightsquigarrow$ charged fermionic zero modes $\lambda_a^i,\ \lambda_b^j$ induce coupling

  $W_{n.p.} \ni Y_\alpha\, Y_\beta\ \mathbf{10}^\alpha\, \mathbf{10}^\beta\, \mathbf{5}_H\ e^{-\frac{\mathrm{Vol_E}}{g_s}}$ if $I_{a,\Xi} = 1 = I_{b,\Xi}$

  Blumenhagen,Cvetič,Lüst,Richter,Weigand 2007

- **Drawback:** realistic GUT models in Type II require
  $S_{\mathrm{inst.}} \simeq \frac{\mathrm{Vol_E}}{g_s} \longrightarrow 0$

  Philosophy: Search for setup where by classical D-terms $Vol_{\mathrm{E}} = 0$
  quantum corrections will resolve this at $Vol_{\mathrm{E}} = \mathcal{O}(l_s)$
  Note: GUT brane can still be large!

Summary of approach

General requirements on compact CY:

- divisor D_a with $h^{(0,1)}(D_a) = 0 = h^{(0,2)}(D_a)$ for GUT brane
- existence of relative two-cycles on D_a for $\mathcal{L}_Y$
- additional divisor D_b with intersection $D_a \cap D_b$
- define orientifold action, preferrably such that $D_{a'} \neq D_a$

SU(5) property	mechanism
no vector-like matter	localisation on curves
1 vector-like of Higgs	choice of line bundles
3-2 splitting	Wilson lines on $g = 1$ curve
3-2 split + no dim=5 p^+-decay	local. of H_u, H_d on disjoint comp.
$\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}_H$ Yukawa	perturb. or D3-instanton
$\mathbf{10}\,\mathbf{10}\,\mathbf{5}_H$ Yukawa	presence of appropriate D3-instanton

Explicit constructions

del Pezzo transitions of quintic $Q = \mathbb{P}^4[5]$, $\quad (h^{1,1} = 1, \quad h^{2,1} = 101)$

see also: Grimm,Klemm '08

1st step in chain of transitions: $Q \to Q^{dP_6}$

- create dP_6 singularity by fixing some complex structure moduli
- blow up singularity by pasting in a dP_6

$\Rightarrow h^{1,1}(Q^{dP_6}) = 2, \quad h^{2,1}(Q^{dP_6}) = 90$

$\Rightarrow$ **no open moduli: dP_6 is rigid** ✓

del Pezzo: $\mathbb{P}^2$ with n points blown up to a $\mathbb{P}^1$ curve E_i,
$H^{1,1}(dP_n) = \langle l, E_1, \ldots E_n \rangle, \quad l \cdot l = 1 = -E_i \cdot E_i$
canonical class $K = \mathcal{O}(f), f = -3l + \sum_i E_i$

$h^{1,1}(Q^{dP_6}) = 2 \Rightarrow$ only f is non-trivial on Q^{dP_6}
trivial ones: those orthogonal to f: $\langle l - E_1 - E_2 - E_3, E_i - E_{i+1} \rangle$

$\Rightarrow$ **ingredients for massless $U(1)_Y$** ✓

$c_1(\mathcal{L}_Y) = E_i - E_j$ leads to no vectorlike exotics from breaking of $\mathbf{24}$

Explicit constructions

toric description to analyse intersection form and topology of divisors

scaling relations: $\{x_i\} \simeq \{\lambda^{Q_1(x_i)} x_i\} \simeq \{\mu^{Q_2(x_i)} x_i\}$

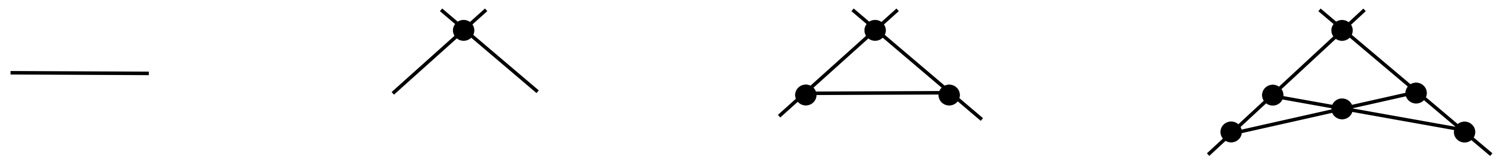
	u_1	u_2	u_3	u_4	u_5	w
Q_1	1	1	1	1	1	0
Q_2	0	0	0	0	1	1
class	H	H	H	H	H +X	X

Q^{dP_6} :

$P_{(5,2)}(u_i, v, w) = 0$

sequence of transitions: fix more compl. structure and blow-up

Q^{dP_6} $\quad Q^{(\mathrm{dP}_7)^2}$ $\quad Q^{(\mathrm{dP}_8)^3}$ $\quad Q^{(\mathrm{dP}_9)^4}$



- new del Pezzos intersect in $\mathbb{P}^1$ $\Rightarrow$ matter curves
- E_6 sublattice of each higher dP_n is trivial on Calabi-Yau $\Rightarrow$ $U(1)_Y$

Explicit constructions

2 types of involutions: inversion: $x_i \to -x_i$ or exchange : $x_i \leftrightarrow x_j$

Focus on exchange involution on $Q^{dP_9^4}$:

	z	u_1	u_2	v_1	v_2	w_1	w_2	x_1	x_2
Q_1	1	1	1	1	1	0	0	0	0
Q_2	0	0	0	0	**1**	**1**	0	0	0
Q_3	0	0	0	**1**	0	0	**1**	0	0
Q_4	0	0	**1**	0	0	0	0	**1**	0
Q_5	0	**1**	0	0	0	0	0	0	**1**
class	D_5	$D_5{+}D_9$	$D_5{+}D_6$	$D_5{+}D_8$	$D_5{+}D_7$	D_7	D_8	D_6	D_9

Involution: $\sigma : v_1 \leftrightarrow v_2, \qquad w_1 \leftrightarrow w_2$

invariant: $v_1 v_2, \quad w_1 w_2, \quad v_1 w_1 + v_2 w_2,$ anti-inv.: $v_1 w_1 - v_2 w_2$

$h^{1,1}_+ = 4, \qquad h^{1,1}_- = 1$

O7-plane: $v_1 w_1 - v_2 w_2 = 0, \qquad [O7] = [D_5 + D_7 + D_8]\ , \quad \chi(O7) = 37$

further: $N_{03} = 3 \Rightarrow N_{03} + \chi(O7) = 40$

Explicit constructions

SU(5) GUT stack on dP_9:

$$U(5): \quad D_a = D_7, \quad D'_a = D_8,$$
$$U(1): \quad D_b = D_5, \quad D'_b = D_5,$$
$$U(3): \quad D_c, = D_5 + D_7, \quad D'_c = D_5 + D_8$$

D7-TAD ✓

matter curves:

$\mathbf{10}$: $D_7 \cap D_8 = \mathbb{P}^1$ $\quad$ $\mathbf{5_m}$: $D_7 \cap D_5 = T^2$ $\quad$ $\mathbf{5_H} + \overline{\mathbf{5}}_\mathbf{H}$: $D_8 \cap D_5 = T^2$

find line bundles + B-field that are quantised properly
(Freed-Witten: divisors are non-Spin!)

possible to obtain exactly 3 generations, 1 vectorlike Higgs pair, no exotics

Drawbacks:

- Overshooting of D3-TAD by 3 units
- uncertainty of one K-theory constraint

Semi-realistic global example

Example on manifold $Q^{dP_9^4}$: GUT brane on dP_9

[Blumenhagen,Braun,Grimm,Weigand 0811.2936]

property	mechanism	status
globally consistent	tadpoles + K-theory	$\checkmark^{*,**}$
D-term susy	vanishing FI-terms inside Kähler cone	$\checkmark$
gauge group $SU(5)$	$U(5) \times U(1)$ stacks	$\checkmark$
3 chiral generations	choice of line bundles	$\checkmark$
no vector-like matter	localisation on $g = 0, 1$ curves	$\checkmark$
5 vector-like Higgs	choice of line bundles	$\checkmark$
no adjoints	rigid 4-cycles, del Pezzo	$\checkmark$
GUT breaking	$U(1)_Y$ flux on trivial 2-cycles	$\checkmark$
3-2 splitting	Wilson lines on $g = 1$ curve	$\checkmark$
3-2 split + no dim=5 p-decay	local. of H_u, H_d on disjoint comp.	–
$\mathbf{10}\,\overline{\mathbf{5}}\,\overline{\mathbf{5}}_{\mathbf{H}}$ Yukawa	perturbative	$\checkmark$
$\mathbf{10}\,\mathbf{10}\,\mathbf{5}_{\mathbf{H}}$ Yukawa	presence of appropriate D3-instanton	$-^{***}$

Another class of geometries

Second type of backgrounds:

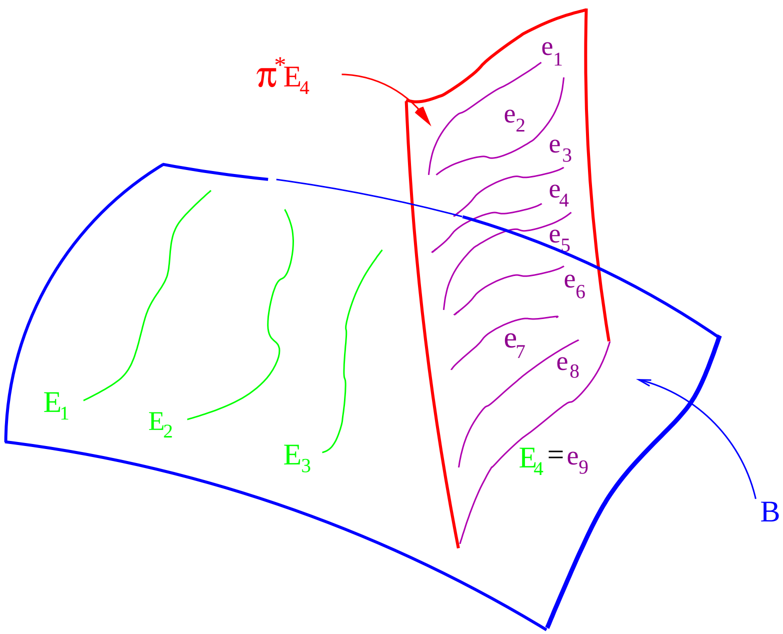
Elliptic fibration over del Pezzo dP_n and flop transistions thereof

$\pi : M_r \to \mathcal{B} = dP_r$

divisors of M_n:

- base $\mathcal{B}$
- ell. fibration over 2-cyles on base

intersecting along elliptic fibre



pullback divisors are dP_9

flops to dP_8 describable in toric language

Swiss cheese type intersection form: $Vol = \Gamma_0^3 - \Gamma_B^3 - \sum_i \Gamma_i^3$

$\Rightarrow$ D-term constraint forces cycles on boundary of Kähler cone

involution of type $x \to -x$ leads to 3-generation examples with instanton generated Yukawas

Conclusions

Type IIB orientifolds suitable arena for SU(5) GUT model building

Recent technological input:

- GUT breaking by $U(1)_Y$ flux as in F-theory
- $\mathbf{10}\,\mathbf{10}\,\mathbf{5_H}$ couplings in Type IIB by exotic D-brane instantons

Realisation of many, but not all phenomenologically desirable features in globally consistent Type IIB orientifolds

Open issues include:

- models with Higgs localised on different curves
- SUSY breaking?

Extension to proper F-theory models on the way via uplift
[Blumenhagen,Grimm,Jurke,T.W. 0906.0013] cf. Talk by R. Blumenhagen

Structure of used manifolds promising for moduli stabilisation:
'swiss cheese' form as in large volume scenario [Conlon, Quevedo et al.]

Dream: Explicit examples with all moduli stabilised!