Dynamics of intersecting brane system

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(K. Maeda, N. Ohta, K. Uzawa, JHEP 06 (2009) 051, arXiv:0903. 5483 [hep-th])
(P. Binetruy, M. Sasaki, K. Uzawa, to appear in PRD)

[1] Introduction

- Time dependent solution of Einstein equations in higher dimensional theory

↓ **target**

- Analysis of the early universe, higher dimensional BH

String theory, supergravity theory : higher dimension (D>4)

↓

How to find our present 4-dim our universe ?

It is necessary to obtain the dynamical solution of the field equations (including the Einstein equations) , and to extract information of the cosmological behavior.

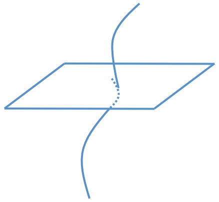
- Dynamics of 4d or internal space

- Classical (p-brane) solution of SUGRA
  (G. Horowitz & A. Strominger; Nucl. Phys.B (1990) 197)

- Super p-brane solution
  (M.J.Duff & J.X.Lu; Nucl. Phys.B 416 (1994) 301)

- Intersecting brane solution : (R. Guven; Phys.Lett.B276, (1992) 49.)

These are classical solutions arise if the gravity are coupled not only to single gauge field but to several combination of the scalar and gauge fields in the supergravity.



- Supersymmetric solution of Intersecting brane
  (G. Papadopoulos & P. K. Townsend; Phys.Lett.B 380 (1996) 273)
  (A. A. Tseytlin, Nucl.Phys.B475 (1996) 149)

- Classification of intersecting brane solutions
  (E. Bergshoeff, et. al. Nucl.Phys.B494 (1997) 119)

☆ Dynamical solution in SUGRA

- 5-dim time dependent solution
  (K. Behrndt & M. Cvetič, Class.Quant.Grav.20 (2003) 4177)

- 10-dimensional IIB model (D3-brane)
  (Gibbons & Lu & Pope; Phys.Rev.Lett. 94 (2005) 131602)

- Heterotic M-theory
  (W. Chen, et al., Nucl.Phys. B732 (2006) 118)

- Time dependent Klebanov & Strassler solution in 10-dim IIB model
  (H. Kodama & K. Uzawa ; JHEP 07 (2005) 061)
  (H. Kodama & K. Uzawa ; JHEP 03 (2006) 053)

- Dynamical p-brane solution
  (P. Binetruy, M.Sasaki, K.Uzawa, arXiv:0712.3615 [hep-th])

- Intersecting p-brane system
  (K. Maeda, N. Ohta, K. Uzawa, JHEP 0409 (2009) 025)

Dynamical solution of 4-dim gravity
(D.Kastor & J. Traschen ; Phys.Rev. D 47 (1993) 5370)

・4-dim Einstein-Maxwell system + cosmological constant

◇ 4-dimensoinal action :

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(R - \frac{1}{4} F_{MN} F^{MN} - 2\Lambda \right)$$

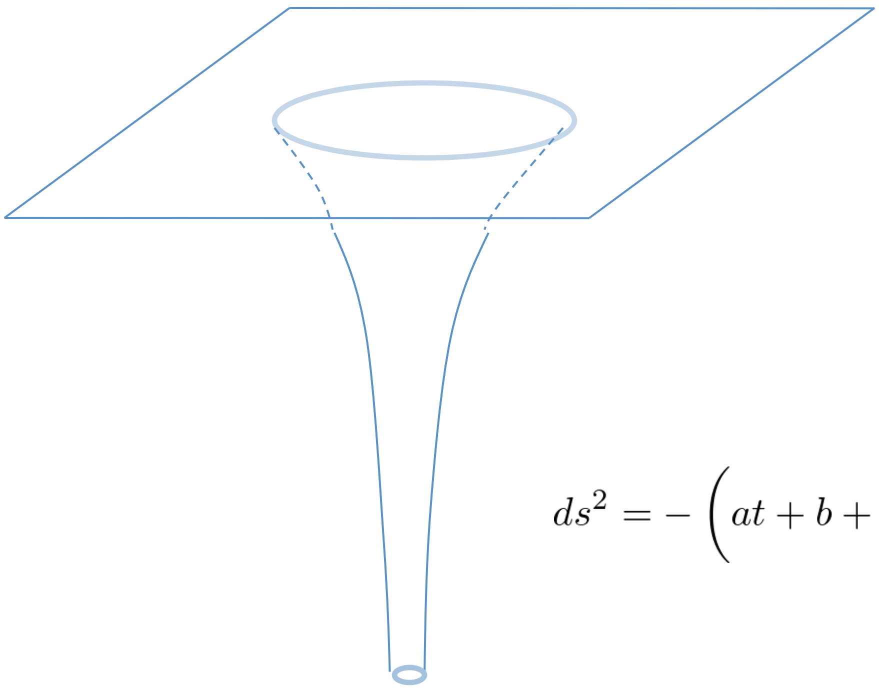
☆ Solution of field equations :

$$ds^2 = -h^{-2}(t,r)dt^2 + h^2(t,r)(dr^2 + r^2 d\Omega_2)$$

$$\begin{aligned} h(t,r) &= at + b + \frac{M}{r}, \\ F_2 &= d\left(h^{-1}\right) \wedge dt, \\ a &= \pm\sqrt{\frac{\Lambda}{3}} \end{aligned}$$

⇒Time dependent solution

☆ Cosmological dynamics ⇒ linear function of time



4-dim de Sitter spacetime

$$r \to \infty, \quad a\tau = \ln(at + b)$$
$$ds^2 = -d\tau^2 + e^{2a\tau}(dr^2 + r^2 d\Omega^2),$$

$$ds^2 = -\left(at + b + \frac{M}{r}\right)^{-2} dt^2 + \left(at + b + \frac{M}{r}\right)^2 (dr^2 + r^2 d\Omega_2)$$

$$r \to 0,$$
$$ds^2 = -\left(\frac{M}{r}\right)^{-2} dt^2 + \left(\frac{M}{r}\right)^2 (dr^2 + r^2 d\Omega^2)$$

4-dim Reissner & Nordstrøm BH

[2] Dynamical solution of p-brane system

(G.W. Gibbons, H. Lu, C.N. Pope Phys.Rev.Lett.94:131602,2005)
(H. Kodama & K. Uzawa ; JHEP 07 (2005) 061)
(P. Binetruy, M. Sasaki, K. Uzawa, arXiv:0712.3615)

Let us consider the case of an arbitrary p-brane background

$$
\begin{aligned}
S &= \frac{1}{2\kappa^2}\int\left(R*\mathbf{1}_D - \frac{1}{2}d\phi\wedge *d\phi - \frac{1}{2}e^{-c\phi}F_{(p+2)}\wedge *F_{(p+2)}\right),\\
c^2 &= 4 - \frac{2(p+1)(D-p-3)}{D-2}.
\end{aligned}
$$

The dynamical background of the p-brane can be written by

$$
\begin{aligned}
ds^2 &= h^{-(D-p-3)/(D-2)}q_{\mu\nu}dx^\mu dx^\nu + h^{(p+1)/(D-2)}u_{ij}dy^i dy^j,\\
e^\phi &= h^{-c/2}, \qquad h(x,y) = h_0(x) + h_1(y),\\
F_{(p+2)} &= d(h^{-1})\wedge\Omega(\mathrm{X}), \qquad \Omega(\mathrm{X}) = \sqrt{-q}\,dx^0\wedge dx^1\wedge\cdots\wedge dx^p
\end{aligned}
$$

In the $c \neq 0$ case, the field equations are reduced to

$$
\begin{aligned}
&R_{\mu\nu}(\mathrm{X}) = 0, \qquad R_{ij}(\mathrm{Y}) = 0,\\
&D_\mu D_\nu h_0 = 0, \qquad \triangle_{\mathrm{Y}} h_1 = 0
\end{aligned}
$$

	0	1	$\cdots$	p	p+1	$\cdots$	D
p-brane	○	○	○	○			

- Internal and external space are Ricci flat.

In the $c = 0$ case, the field equations are reduced to

$$R_{\mu\nu}(\mathrm{X}) = 0, \qquad R_{ij}(\mathrm{Y}) = \frac{b}{2}\lambda u_{ij}(\mathrm{Y}),$$
$$D_\mu D_\nu h_0 = \lambda q_{\mu\nu}(\mathrm{X}), \qquad \triangle_{\mathrm{Y}} h_1 = 0.$$

◆ For example, in the case of $q_{\mu\nu} = \eta_{\mu\nu}, \quad u_{ij} = \delta_{ij}$
the solution is

$$(1)\ c \neq 0 \quad : \qquad h_0(x) = c_\mu x^\mu + \tilde{c}, \quad h_1(y) = \sum_l \frac{M_l}{|y^i - y^i_l|^{D-p-3}}$$

(G.W. Gibbons, H. Lu, C.N. Pope; Phys.Rev.Lett.94:131602,2005)

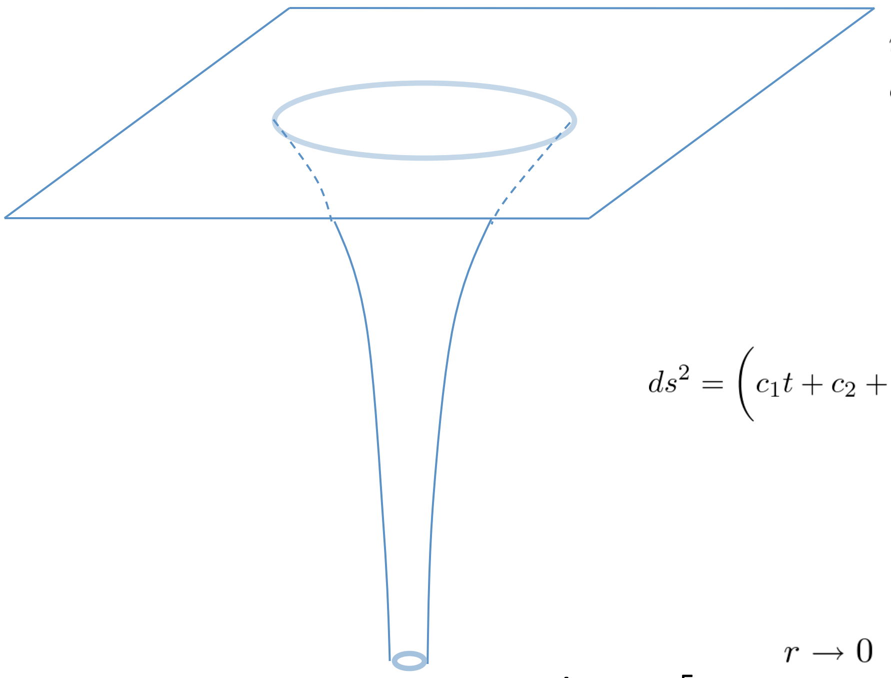
$$(2)\ c = 0 \quad : \qquad h_0(x) = \frac{\lambda}{2}x^\mu x_\mu + c_\mu x^\mu + \tilde{c}, \quad h_1(y) = \sum_l \frac{M_l}{|y^i - y^i_l|^{D-p-3}}$$

(H. Kodama & K. Uzawa ; JHEP 07 (2005) 061)
(P. Binetruy, M.Sasaki, K.Uzawa, arXiv:0712.3615 [hep-th])

$c_\mu, \quad \tilde{c}$: constant parameters

$$ds^2 = h^{-(D-p-3)/(D-2)}\eta_{\mu\nu}dx^\mu dx^\nu + h^{(p+1)/(D-2)}(dr^2 + r^2 d\Omega^2),$$
$$h(t,r) = c_1 t + c_2 + M r^{-D+p+3}$$

For example, $D = 10, \quad p = 3$



$r \to \infty,$

$$ds^2 = (c_1 t + c_2)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + (c_1 t + c_2)^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

$$ds^2 = \left(c_1 t + c_2 + \frac{M}{r^4}\right)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + \left(c_1 t + c_2 + \frac{M}{r^4}\right)^{1/2}(dr^2 + r^2 d\Omega_5^2)$$

$r \to 0$

$$ds^2 = \left(\frac{r}{M}\right)^2 \eta_{\mu\nu}dx^\mu dx^\nu + \left(\frac{M}{r}\right)^2 dr^2 + d\Omega_5^2$$

★ Klebanov & Strassler soltuion (H. Kodama & K. Uzawa ; JHEP 07 (2005) 061)

- Calabi-Yau space (conifold compactification)
- constant dialton

$$ds^2 = h^{-1/2}(x,r)\eta_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(x,r)(x,r)(dr^2 + r^2\, ds^2(\mathrm{T}^{11}))$$

$$*_Y G_3 = iG_3$$

$$\tilde{F}_5 = (1 \pm *)d(h^{-1}) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

- Compactification of the static solution was discussed by Giddings, Kachru and Polchinski (Phys.Rev.D66:106006,2002)

◆ Time dependent solution of IIB SUGRA

☆ Cosmological dynamics ⇒ linear function of time

$$h(x,r) = a_\mu x^\mu + b + \frac{2L^4 + (g_s M)^2\{\ln(r/r_0) + 1/4)\}}{2r^4}$$

[3] Dynamical solution of intersecting p-brane system

(K. Maeda, N. Ohta, K. Uzawa: arXiv 0903 5483[hep-th]])

D-dim action

$$S = \frac{1}{2\kappa^2}\int \left[R * \mathbf{1}_D - \frac{1}{2} d\phi \wedge * d\phi - \sum_I \frac{1}{2(p_I+2)!} e^{c_I \phi} F_{(p_I+2)} \wedge * F_{(p_I+2)} \right],$$

$$c_I^2 = 4 - \frac{2(p_I+1)(D-p_I-3)}{D-2}$$

Ansatz for the dynamical background

$$\begin{aligned} ds^2 &= -B(t,y)dt^2 + \sum_{\mu=1}^{p} C^\mu(t,y)(dx^\mu)^2(\mathrm{X}) + A(t,y)u_{ij}(\mathrm{Y})dy^i dy^j \\ &= -\prod_I h_I^a(t,y)dt^2 + \sum_{\mu=1}^{p}\prod_I h_I^{\frac{\delta_I^\mu}{D-2}}(t,y)(dx^\mu)^2(\mathrm{X}) + \prod_I h_I^b(t,y)u_{ij}(\mathrm{Y})dy^i dy^j, \end{aligned}$$

$$a = -\frac{D-p_I-3}{D-2}, \qquad b = \frac{p_I+1}{D-2}, \qquad \delta_I^\mu = \begin{cases} -(D-p_I-3) & \text{for } \mu \parallel I \\ p_I+1 & \text{for } \mu \perp I \end{cases}$$

$$e^\phi = \prod_I h_I^{\epsilon_I c_I/2}, \qquad \epsilon_I = \pm 1$$

$$F_{(p_I+2)} = d(h_I^{-1}) \wedge \Omega(\mathrm{X}_I) \qquad \Omega(\mathrm{X}_I) = dt \wedge dx^{p_1} \wedge \cdots \wedge dx^{p_I}$$

Let us assume

$$B^{1/2}\left(\sum_{\mu} C^{\mu}\right)^{1/2} A^{(D-p-3)/2}=1, \qquad e^{\epsilon_I c_I \phi}=B\left(\prod_{\mu||I} C^{\mu}\right) h_I^2$$

Then field equations then reduce to

$$\sum_I (p_I+1)(\partial_t \ln h_I)^2-\sum_I\sum_{I'}(p_{I'}+1)\partial_t \ln h_I \partial_t \ln h_{I'}=0,$$

$$\sum_I \delta_I^{\mu}(\partial_t \ln h_I)^2-\sum_I\sum_{I'}\delta_{I'}^{\mu}\partial_t \ln h_I \partial_t \ln h_{I'}=0$$

$$R_{ij}(\mathrm{Y})=0,$$

$$h_I(t,y)=H_I(t)+K_I(y), \quad \partial_t^2 H_I=0, \quad \triangle_{\mathrm{Y}} K_I=0,$$

Above equations can be satisfied it only if there is only one function h_I depending on both y^i and t .

As a special example, we consider the case $u_{ij} = \delta_{ij}$

In this case, the solution for h_I can be obtained explicitly as

$$h_I(t,y) = At + B + \sum_\alpha \frac{M_{\alpha,I}}{|y^i - y^i_\alpha|^{D-p-3}},$$

where $A,\ B$ and M^I_α are constant parameters.

Let us consider the dynamics of 4-dim universe. To find an expanding universe, we have to smear and compactify the bulk space as well as the brane world volume (fixing our universe at some position in Z space).

	branes	dim(Z)	$s_{\tilde{I}}$ or $s_{\tilde{\text{I}}}$	$\beta_{\tilde{I}}$ or $\beta_{\tilde{\text{I}}}$	$\beta^{(1)}_{\tilde{I}}$ or $\beta^{(1)}_{\tilde{\text{I}}}$	$\beta^{(2)}_{\tilde{I}}$ or $\beta^{(2)}_{\tilde{\text{I}}}$	$\beta^{(3)}_{\tilde{I}}$ or $\beta^{(3)}_{\tilde{\text{I}}}$
case 1 ($\tilde{I}$ =M5)	M2-M5	4	$-1/3$	$-1/5$	0	1/7	1/4
	M5-M5	3	0	0	1/7	1/4	–
	M5-M5-M5	1	2/3	1/4	–	–	–
	M2-M5-M5	3	0	0	1/7	1/4	–
	M2-M2-M5	3	0	0	1/7	1/4	–
	M2-M2-M5-M5	3	0	0	1/7	1/4	–
case 2 ($\tilde{I}$ =M2)	M2-M5	4	$-7/6$	$-1/5$	0	1/7	1/4
	M2-M5-M5	3	-1	0	1/7	1/4	–
	M2-M2-M5	3	-1	0	1/7	1/4	–
	M2-M2-M5-M5	3	-1	0	1/7	1/4	–

Even for the fastest expanding case, the power is too small to give a realistic expansion law such as that in the matter dominated era $a \propto \tau^{2/3}$ or that in the radiation dominated era $a \propto \tau^{1/2}$.

☆ Compactifying all brane volume :

ex.) M2-M2-M5-M5-brane

	0	1	2	3	4	5	6	7	8	9	10
M2	○				○		○				
M2	○					○		○			
M5	○	○	○	○	○	○					
M5	○	○	○	○			○	○			

In terms of the cosmic time $\tau = \frac{4}{3} t_0 (t/t_0)^{3/4}$, 4-dim metric is given by

$$ds_4^2 = -\left(h_2 H_2 H_5 H'_5\right)^{-1/2} d\tau^2 + a^2(\tau) \left(h_2 H_2 H_5 H'_5\right)^{1/2} \left(dr^2 + r^2 d\Omega_2^2\right),$$

$$h_2 = 1 + \frac{M_2(\tau)}{r}, \quad a(\tau) \equiv \left(\frac{\tau}{\tau_0}\right)^{1/3}, \quad M_2 = \left(\frac{\tau}{\tau_0}\right)^{-4/3} Q_2, \quad \tau_0 \equiv \frac{4}{3} t_0$$

If r→∞, the line element becomes

$$ds^2 = -d\tau^2 + a^2(\tau)(dr^2 + r^2 d\Omega_2^2)$$

Hence, the solution approaches asymptotically FRW universe with scale factor. Since M2-M2-M5-M5-brane solution gives a BH, this is a BH in the expanding Universe.

◆ Intersections of KK monopoles and KK waves in eleven dimensions

▪In D-dimensional spacetime, we can obtain the electric 0-brane and the magnetic (D-4)-brane after compactification on one direction of (D+1)-dimensional theory. These objects corresponds to the KK-wave and KK-monopole in (D+1)-dimensions respectively.

(i) KK-wave in eleven-dimension: uplift 0-brane in 10-dimension

$$ds^2_{\rm W} = -dt^2 + dz^2 - H(dt - dz)^2 + u_{ij}({\rm Y})dy^i dy^j$$

$$H(t, y) = H_0(t) + H_1(y), \quad \partial_t^2 H_0 = 0, \quad \triangle_{\rm Y} H_1 = 0$$

(ii) KK-monopole in eleven-dimension: uplift 6-brane in 10-dimension

$$ds^2_{\rm M} = q_{\mu\nu}dx^\mu dx^\nu + h^{-1}(dz + A_i dy^i)^2 + hu_{ij}({\rm Y})dy^i dy^j,$$

$$h(x, y) = h_0(x) + h_1(y), \quad D_\mu D_\nu h_0 = 0, \quad \triangle_{\rm Y} h_1 = 0,$$

where the gauge potential A_i is given by

$$\partial_i A_j - \partial_j A_i = -\epsilon_{ijk}\partial^k h$$

[4] Lower dimensional effective theory

(H. Kodama & K. Uzawa ; JHEP 03 (2006) 053)

★(p+1)-dimensional effective theory (No flux case)

$$S = \frac{1}{2\kappa^2}\int\left(R * \mathbf{1}_D - \frac{1}{2} d\phi \wedge * d\phi\right)$$

★ Ansatz for background

$$\begin{aligned} ds^2 &= h_0^a(x)\, ds^2(\mathrm{X}) + h_0^b(x)\, ds^2(\mathrm{Y}), \\ e^\phi &= h_0^{-c/2} \end{aligned}$$

$$a = -\frac{D-p-3}{D-2}, \qquad b = \frac{p+1}{D-2}$$

★ Field equations are reduced to

$$\begin{aligned} &R_{\mu\nu}(\mathrm{X}) - h_0^{-1} D_\mu D_\nu h_0 = 0, \\ &R_{ij}(\mathrm{Y}) = 0 \end{aligned}$$

□ lower-dimensional effective action

- No flux and internal space is Ricci flat space
- Scalar field satisfies the equation of motion.

★ Ansatz for background

$$
\begin{aligned}
ds^2 &= h_0^a(x)\, ds^2(\mathrm{X}) + h_0^b(x)\, ds^2(\mathrm{Y}), \\
e^{\phi} &= h_0^{-c/2}
\end{aligned}
$$

$$
\begin{aligned}
S &= \frac{1}{2\tilde{\kappa}^2} \int_{\mathrm{X}} h_0(x) R(\mathrm{X}) *_{\mathrm{X}} \mathbf{1}_{(p+1)}, \\
\tilde{\kappa} &= \left[V_{(D-p-1)}\right]^{-1/2} \kappa, \qquad V_{(D-p-1)} = \int * \mathbf{1}_{(D-p-1)}
\end{aligned}
$$

□ (p+1)-dimensional field equations

$$
\begin{aligned}
&R_{\mu\nu}(\mathrm{X}) = h_0^{-1} D_\mu D_\nu h_0, \\
&\triangle_{\mathrm{X}} h_0 = 0
\end{aligned}
$$

(p+1)-dimensional effective theory with Flux

◇ D-dimensional model

● Ansatz for background

$$
\begin{aligned}
ds^2 &= h^{-(D-p-3)/(D-2)}(x,y)q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + h^{(p+1)/(D-2)}(x,y)u_{ij}(\mathrm{Y})dy^i dy^j, \\
e^\phi &= h^{c/2}, \qquad h(x,y) = h_0(x) + h_1(y), \\
F_{(p+2)} &= d(h^{-1}) \wedge \Omega(\mathrm{X}_{p+1}), \qquad \Omega(\mathrm{X}_{p+1}) = \sqrt{-q}dx^0 \wedge dx^1 \wedge \cdots \wedge dx^p
\end{aligned}
$$

- Internal space is Einstein space
- Gauge fields satisfy field equations.

◎ D-dimensional action

$$
\begin{aligned}
S &= \frac{1}{2\kappa^2}\int\left(R * \mathbf{1}_D - \frac{1}{2}d\phi \wedge *d\phi - \frac{1}{2}e^{-c\phi}F_{(p+2)} \wedge *F_{(p+2)}\right), \\
c^2 &= 4 - \frac{2(p+1)(D-p-3)}{D-2}.
\end{aligned}
$$

☐ (p+1)-dimensional effective action $(c \neq 0)$

- Internal space is Ricci flat space

$$
\begin{aligned}
S &= \frac{1}{2\tilde{\kappa}^2}\int_{\mathrm{X}} H(x) R(\mathrm{X}) *_{\mathrm{X}} \mathbf{1}_{(p+1)}, \\
H(x) &= h_0(x) + \bar{c}, \quad \bar{c} = V_{(D-p-1)}^{-1}\int_{\mathrm{Y}} h_1 *_{\mathrm{Y}} \mathbf{1}_{(D-p-1)}, \\
\tilde{\kappa} &= \left[V_{(D-p-1)}\right]^{-1/2}\kappa, \quad V_{(D-p-1)} = \int_{\mathrm{Y}} *_{\mathrm{Y}}\mathbf{1}_{(D-p-1)}
\end{aligned}
$$

Conformal transformation : $ds^2(\mathrm{X}) = H^{2/(1-p)} ds^2(\bar{\mathrm{X}})$

$$
S = \frac{1}{2\tilde{\kappa}^2}\int_{\bar{\mathrm{X}}}\left[R(\bar{\mathrm{X}}) *_{\bar{\mathrm{X}}} \mathbf{1}_{(p+1)} - \frac{p}{(p-1)} d\ln H \wedge *_{\bar{\mathrm{X}}} d\ln H\right]
$$

★ Field equations

$$
\begin{aligned}
R_{\mu\nu}(\mathrm{X}) &= H^{-1} D_\mu D_\nu H, \\
\triangle_{\mathrm{X}} H &= 0
\end{aligned}
$$

Lower-dimensional theory

⟷

$$
R_{\mu\nu}(\mathrm{X}) = 0, \quad D_\mu D_\nu h_0 = 0
$$

Higher-dimensional theory

Solution: G. Shiu, et al., JHEP06(2008)024

☐ (p+1)-dimensional effective action $(c=0)$

▪ Internal space is Einstein space

$$
\begin{aligned}
S &= \frac{1}{2\tilde{\kappa}^2}\int_{\mathrm{X}}\left[H(x)R(\mathrm{X})+\frac{\lambda}{2}b(p+1)(D-p-1)\right]*_{\mathrm{X}}\mathbf{1}_{(p+1)},\\
H(x) &= h_0(x)+\bar{c}; \quad \bar{c}:=V_{(D-p-1)}^{-1}\int_{\mathrm{Y}}h_1 *_{\mathrm{Y}}\mathbf{1}_{(D-p-1)},\\
\tilde{\kappa} &= \left[V_{(D-p-1)}\right]^{-1/2}\kappa, \quad V_{(D-p-1)}=\int_{\mathrm{Y}}*_{\mathrm{Y}}\mathbf{1}_{(D-p-1)}
\end{aligned}
$$

Conformal transformation : $ds^2(\mathrm{X}) = H^{2/(1-p)}ds^2(\bar{\mathrm{X}})$

$$
\begin{aligned}
S = \frac{1}{2\tilde{\kappa}^2}\int_{\bar{\mathrm{X}}}\Big[&R(\bar{\mathrm{X}})*_{\bar{\mathrm{X}}}\mathbf{1}_{(p+1)}-\frac{p}{(p-1)}d\ln H\wedge *_{\bar{\mathrm{X}}}d\ln H\\
&+\frac{\lambda}{2}(p+1)(D-p-1)bH^{(1+p)/(1-p)}\Big]
\end{aligned}
$$

★ Field equations

$$
\begin{aligned}
R_{\mu\nu}(\mathrm{X}) &= H^{-1}\left[D_\mu D_\nu H-\frac{\lambda}{4p}(p+1)(D-p-1)bq_{\mu\nu}(\mathrm{X})\right],\\
\triangle_{\mathrm{X}}H &= \frac{(p+1)^2}{4p}(D-p-1)\lambda b.
\end{aligned}
$$

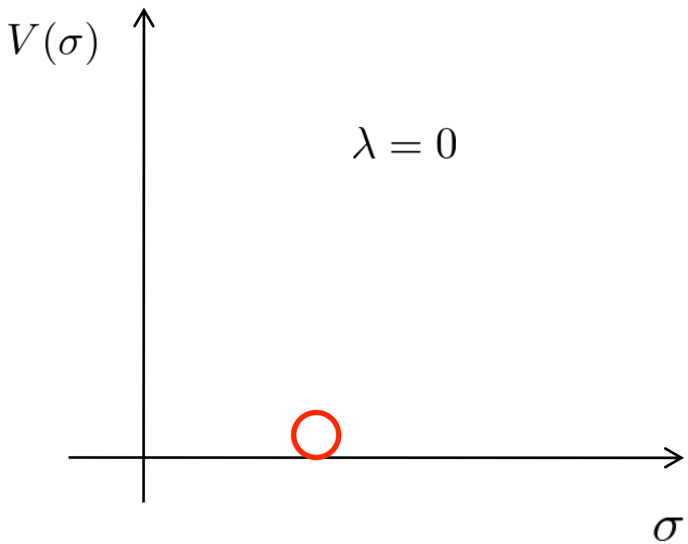
$$
\begin{aligned}
R_{\mu\nu}(\bar{\mathrm{X}}) &= \frac{p}{(p-1)}\bar{D}_\mu\ln H\,\bar{D}_\nu\ln H\\
&\quad-\frac{\lambda}{2(p-1)}(p+1)(D-p-1)bH^{(1+p)/(1-p)}q_{\mu\nu}(\bar{\mathrm{X}}),\\
\triangle_{\bar{\mathrm{X}}}\ln H &= \frac{(p+1)^2}{4p}(D-p-1)\lambda bH^{(1+p)/(1-p)}
\end{aligned}
$$

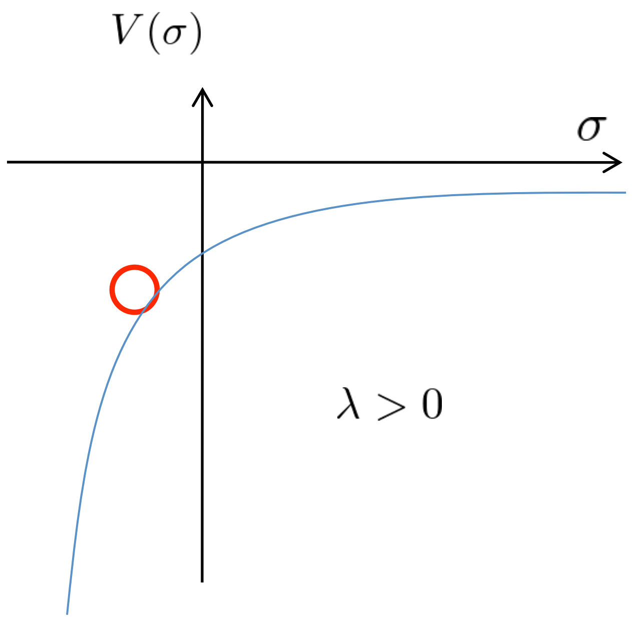
☆ p=3, D=10 case, ⇒ 4-dimensional moduli potential (Einstein frame)

$$
\begin{aligned}
V(\sigma) &= -6\lambda e^{-2\sigma/\sqrt{3}}, \\
\sigma &= \sqrt{3}\ln\left[h_0(x) + V_6^{-1}\int_{\mathrm{Y}} h_1 *_{\mathrm{Y}} \mathbf{1}\right]
\end{aligned}
$$

$\lambda = 0 \cdots$ flat potential,

$\lambda \neq 0 \cdots$ run away potential





[5] Summary :

☆ We give some dynamical intersecting brane solutions with/without M-waves and Kaluza-Klein monopoles in eleven-dimensional supergravity.

☆ We apply these solutions to cosmology and black holes. It is shown that these give FRW cosmological solutions. If we regard the bulk space as our universe, we may interpret them as black holes in the expanding universe.

★ The solutions we found have the property that they are genuinely higher-dimensional in the sense that one can never neglect the dependence on say of warp factor.

★ We also found lower-dimensional effective theories may give us some solutions which are inconsistent with the higher-dimensional Einstein equations.