Heterotic orbifold GUTs in 6D and $K3$ moduli fields

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Outline

The standard model of particle physics

The SM is a particular quantum field theory:

1. It has local gauge symmetry.
2. Matter is described by chiral fermions.
3. Gauge symmetry is broken by the Higgs boson vev.
4. Higgs-matter couplings induce mass terms.

These principles are not very restrictive. Nature chooses

$G_{\mathrm{SM}} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$,

Higgs: $\underbrace{(\mathbb{1},\mathbf{2})_{1/2}}_{h}$,

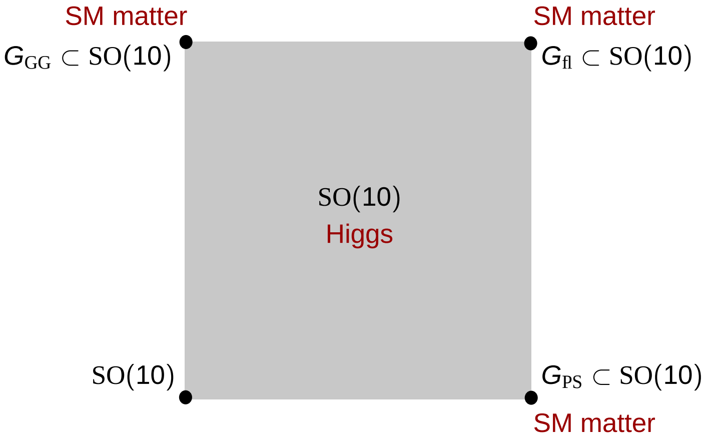
Leptons: $\underbrace{(\mathbb{1},\mathbf{2})_{-1/2}}_{l} + \underbrace{(\mathbb{1},\mathbb{1})_{1}}_{e^c}$,

Quarks: $\underbrace{(\mathbf{3},\mathbf{2})_{1/6}}_{q} + \underbrace{(\overline{\mathbf{3}},\mathbb{1})_{-2/3}}_{u^c} + \underbrace{(\overline{\mathbf{3}},\mathbb{1})_{1/3}}_{d^c}$,

at accessible energies $\lesssim 10^2$ GeV. And beyond?

[Asaka, Buchmüller, Covi 2001]

A 6D orbifold GUT example: The ABC model



Bottom-up $\longrightarrow$ 6D GUT model $\longleftarrow$ Top-down ?

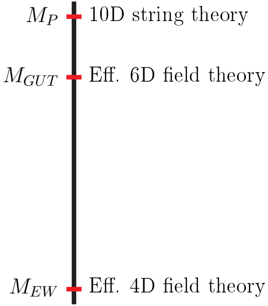
Question: Higher-dimensional orbifold GUTs from string theory?

There is a hierarchy of scales:

String theory $\supset$ GUT $\supset$ Standard model

$M_P = 10^{19}$ GeV — $M_{\mathrm{GUT}} = 10^{16}$ GeV — $M_{\mathrm{EW}} = 10^{2}$ GeV

This may be related to **anisotropic** compact internal dimensions:



A local GUT in 6D

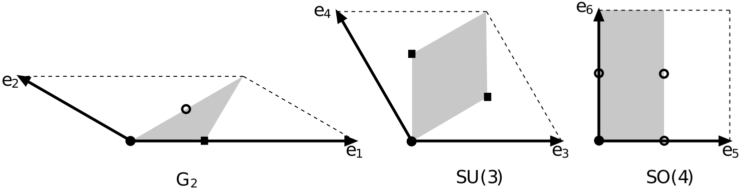
[Buchmüller, Lüdeling, JS 2007]
[Buchmüller, JS 2008]
[JS 2009 (to appear)]

The orbifold target space

Consider the following **orbifold geometry** as target space of three internal complex dimensions ($i = 1, 2, 3$):

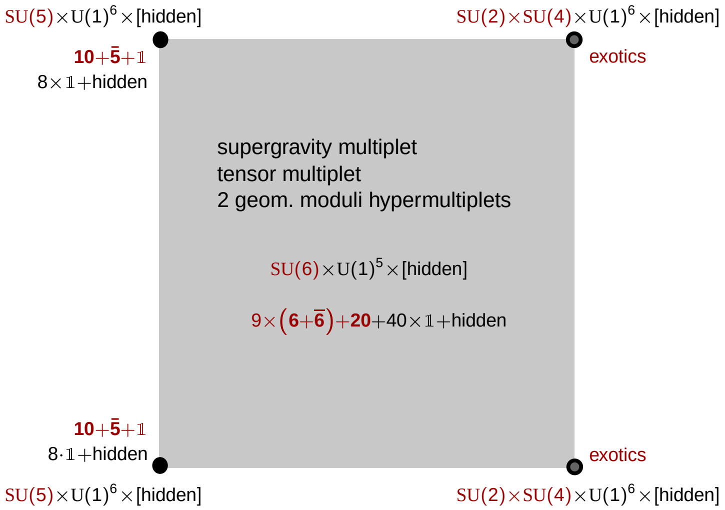
[Kobayashi, Raby, Zhang 2004]

$$\mathcal{M}_6 = \mathbb{C}^3/S \equiv \frac{\mathbb{C}/\Gamma_{\mathrm{G}_2} \times \mathbb{C}/\Gamma_{\mathrm{SU(3)}} \times \mathbb{C}/\Gamma_{\mathrm{SO(4)}}}{\mathbb{Z}_6}$$



Assume a specific gauge embedding V_g, with two Wilson lines:
One WL in $\mathrm{SU}(3)$-plane, one WL in $\mathrm{SO}(4)$-plane,
as in [Buchmüller, Hamaguchi, Lebedev, Ratz 2006].

The effective orbifold GUT in 6D



Unique $\mathrm{U}(1)_X$ and $\mathrm{U}(1)_{B-L}$

The model has local $\mathrm{SU}(5)$ GUT structure:

$$W = \underbrace{C_{ij}^{(u)}\mathbf{10}_{(i)}\mathbf{10}_{(j)}H_u + C_{ij}^{(d)}\bar{\mathbf{5}}_{(i)}\mathbf{10}_{(j)}H_d}_{\text{Yukawa couplings}} + \underbrace{C_{ijk}^{(R)}\bar{\mathbf{5}}_{(i)}\mathbf{10}_{(j)}\bar{\mathbf{5}}_{(k)}}_{\text{dim. 4 proton decay}} + \dots$$

- Symmetry solution: $\mathrm{SU}(5) \times \mathrm{U}(1)_X \subset \mathrm{SO}(10)$.

$$t_X(\mathbf{10}) = \frac{1}{5}\,, \quad t_X(\bar{\mathbf{5}}) = -\frac{3}{5}\,, \quad t_X(H_u) = -\frac{2}{5}\,, \quad t_X(H_d) = \frac{2}{5}\,.$$

- This implies $\mathrm{U}(1)_{B-L}$: $t_{B-L} = t_X + \frac{4}{5}\,t_Y$.
- There is a **unique** embedding at the GUT fixed points:

$$\boxed{\mathrm{SU}(5) \times \mathrm{U}(1)_X \subset \mathrm{SU}(5) \times \mathrm{U}(1)^6}$$

Matter vs. Higgs vs. Exotics

- **Gauge group**: Intersection of local gauge groups

$$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times \underbrace{\mathrm{U}(1)^8}_{\text{Higgsed}} \times [\text{hidden}]$$

- Find four $\mathrm{SU}(5)$ families:

$\mathbf{10}_{(1)}$:	twisted,	localized	$\bar{\mathbf{5}}_{(1)}$:	twisted,	localized
$\mathbf{10}_{(2)}$:	twisted,	localized	$\bar{\mathbf{5}}_{(2)}$:	twisted,	localized
$\mathbf{10}_{(3)}$:	untwisted,	bulk	$\bar{\mathbf{5}}_{(3)}$:	twisted,	6D bulk
$\mathbf{10}_{(4)}$:	untwisted,	bulk	$\bar{\mathbf{5}}_{(4)}$:	twisted,	6D bulk

Zero modes: 3 standard model families.

- Higgs ambiguity:

	H_u-candidates:			H_d-candidates:	
$\mathbf{5}$:	untwisted,	bulk	$\bar{\mathbf{5}}$:	untwisted,	bulk
$\mathbf{5}_1$:	twisted,	6D bulk	$\bar{\mathbf{5}}_1$:	twisted,	6D bulk

- Exotics: $7 \times \mathbf{5}$, $7 \times \bar{\mathbf{5}}$, Require $W \supset M\, \mathbf{5}\bar{\mathbf{5}}$

String selection rules for interactions

The underlying string theory implies rules for superpotential terms

$$W \supset \alpha\, \phi_1 \cdots \phi_M\,.$$

They can be interpreted as symmetries of the effective field theory:

$$G = G_{\text{gauge}} \times G_{\text{discrete}}\,,$$

$$G_{\text{discrete}} = \underbrace{\tilde{\mathbb{Z}}_6^{R^1} \times \tilde{\mathbb{Z}}_3^{R^2} \times \tilde{\mathbb{Z}}_2^{R^3} \times \mathbb{Z}_6^{\text{twist}}}_{\text{discrete } R\text{-symmetry}} \times \underbrace{\mathbb{Z}_3^{\mathrm{SU}(3)} \times \mathbb{Z}_2^{\mathrm{SO}(4)} \times \mathbb{Z}_2^{\mathrm{SO}(4)'}}_{\text{localization symmetry}}\,.$$

The localization symmetry for the G_2-plane acts only on $W_0 \subset W_{\text{tot}}$:

$$W_{\text{tot}} = \underbrace{W_0}_{G\times\mathbb{Z}_6^{G_2}} + \underbrace{W}_{G}\,,$$

W_0 : All ϕ_i from T_1/T_5, or T_2/T_4, or T_3,

W : Mixed terms.

The superpotential

$$G = \underbrace{\mathrm{U}(1)^6}_{\mathbf{Q}} \times \underbrace{\tilde{\mathbb{Z}}_6^{R^1} \times \tilde{\mathbb{Z}}_3^{R^2} \times \tilde{\mathbb{Z}}_2^{R^3} \times \mathbb{Z}_6^{\text{twist}} \times \mathbb{Z}_3^{\mathrm{SU}(3)} \times \mathbb{Z}_2^{\mathrm{SO}(4)} \times \mathbb{Z}_2^{\mathrm{SO}(4)'}}_{\mathcal{K}}$$

$W = \underbrace{s_1 \cdots s_N}_{\lambda} \Phi$ allowed, if $\mathbf{Q}(\lambda\Phi) = 0, \quad \mathcal{K}(\lambda\Phi) = \mathcal{K}_{\text{vac}}.$

Write $\lambda = \omega_0 \lambda_0^\Phi \lambda_s^\Phi,$ $\quad \mathbf{Q}(\omega_0) = 0, \quad \mathbf{Q}(\lambda_0^\Phi) = \mathbf{Q}(\lambda_s^\Phi \Phi) = 0,$

$$\mathcal{K}(\omega_0) = 0, \quad \mathcal{K}(\lambda_0^\Phi) = \mathcal{K}_{\text{vac}} - \mathcal{K}(\lambda_s^\Phi \Phi).$$

Find **basis monomials** of $\ker \mathbf{Q} \cap \ker \mathcal{K}$!

$$W = \mathcal{P}_{\mathbb{N}} \left(\sum_{\omega_0} \omega_0 \right) \left(\sum_{\Phi} \lambda_0^\Phi \lambda_s^\Phi \Phi \right)$$

Partial gauge-Higgs unification

Consider the μ-term and **partial gauge-Higgs unification**:

$$W \supset \mu\, H_u H_d\,, \qquad \begin{cases} H_u = \mathbf{5} & \text{untwisted, } \mathbf{35} = \mathbf{24} + \mathbf{5} + \bar{\mathbf{5}} + \mathbb{1}, \\ H_d = \bar{\mathbf{5}}_1 & \text{twisted, 6D bulk.} \end{cases}$$

Subsequent addition of singlets leads to a maximal vacuum with

- $\mu = 0$ to all orders,
- $\langle W \rangle = 0$ to all orders,
- unbroken matter parity $\mathrm{U}(1)_X \to P_X$,
- decoupled exotics,
- a heavy top-quark, $Y_{tt}^{(u)} \sim g$,

given by $\mathcal{S} = \{ \underbrace{X_0, \bar{X}_0^c, \bar{X}_1, X_1^c, \bar{Y}_2, Y_2^c}_{\text{twisted, 6D bulk}}, \underbrace{U_1^c, U_2, U_3, U_4}_{\text{untwisted}}, \underbrace{S_2, S_5, S_6, S_7}_{\text{twisted, localized}} \}.$

Unbroken discrete symmetries

- The result $\mu = \langle W \rangle = 0$ can be understood by **unbroken discrete symmetries**:

$$\mathrm{U}(1)^6 \times G_{\mathrm{discrete}} \longrightarrow G_{\mathrm{vac}}\left(\mathcal{S}\right) = \left[\tilde{\mathbb{Z}}_4 \times \mathbb{Z}_2\right]_R \times \mathbb{Z}_{60}^X .$$

- The generators have a complicated embedding:

$$t_{[\tilde{\mathbb{Z}}_4]_R} = \left(\frac{1}{2}, 0, -\frac{1}{12}, \frac{5}{8}, \frac{1}{24}, -\frac{1}{30}\right) \times \mathbf{Q} + \frac{1}{2} R^1$$

The μ-term and $\langle W \rangle$ are forbidden by
a shifted discrete R-symmetry.

- The Yukawa couplings for $\mathcal{S}$ are semi-realistic:

$$Y^{(u)} = \begin{pmatrix} s^4 & s^4 & s^5 \\ s^4 & s^4 & s^5 \\ s^5 & s^5 & g \end{pmatrix}, \quad Y^{(d)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ s^1 & s^1 & s^2 \end{pmatrix}, \quad Y^{(l)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ s^{10} & s^{10} & s^6 \end{pmatrix}.$$

$$\frac{T^4/\mathbb{Z}_3 \times T^2}{\mathbb{Z}_2} \quad \text{versus} \quad \frac{K3 \times T^2}{\mathbb{Z}_2}$$

[Work in progress]

$K3$ gauge background

- Here only first step: Matching $K3$ moduli and orbifold fields.
- $K3$ has Euler characteristic 24:

$$\frac{1}{16\pi^2}\int_{K3} \mathrm{tr}\, R^2 = 24$$

- Tadpole cancelation condition:

$$\int_{K3} \left(\mathrm{tr}\, R^2 - \mathrm{tr}\, F^2\right) = 0$$

$\Rightarrow \quad \mathrm{E}_8 \times \mathrm{E}_8$ broken by 24 instantons.

- For comparison with the orbifold result, we consider

Visible sector: $\quad \mathrm{E}_8 \supset \mathrm{SU}(6) \times \underbrace{\langle\mathrm{SU}(2)\rangle}_{\text{6 instantons}} \times \underbrace{\langle\mathrm{SU}(3)\rangle}_{\text{6 instantons}},$

Hidden sector: $\quad \mathrm{E}_8 \supset \mathrm{SO}(8) \times \underbrace{\langle\mathrm{SO}(8)\rangle}_{\text{12 instantons}}.$

$K3$ moduli space

Gauge bundle moduli:

$\mathrm{SU}(2)$ gauge bundle:	9 hypermultiplets,
$\mathrm{SU}(3)$ gauge bundle:	10 hypermultiplets,
$\mathrm{SO}(8)$ gauge bundle:	44 hypermultiplets.

Geometrical moduli:

$$\frac{O(4,20)}{O(4) \times O(20)} \quad \Rightarrow \quad 20 \text{ hypermultiplets.}$$

Total:

$$K3: \quad \underbrace{19}_{\text{vis. } \mathrm{E}_8} + \underbrace{44}_{\text{hid. } \mathrm{E}_8} + \underbrace{20}_{\text{geom.}} = 83 \ \text{ moduli}\,.$$

Spectra

- $K3$ spectrum, $G = \mathrm{SU}(6) \times \mathrm{SO}(8)$: [cf. Bershadsky et al. 1996]

$$(\mathbf{20}, \mathbb{1}) + 9 \times \left[(\mathbf{6}, \mathbb{1}) + (\overline{\mathbf{6}}, \mathbb{1})\right] + 4 \cdot [(\mathbb{1}, \mathbf{8}) + (\mathbb{1}, \mathbf{8}_s) + (\mathbb{1}, \mathbf{8}_c)]$$

- $T^4/\mathbb{Z}_3$ spectrum, $\mathrm{SU}(6) \times \mathrm{U}(1)^3 \times \left[\mathrm{SU}(3) \times \mathrm{SO}(8) \times \mathrm{U}(1)^2\right]$:

$$\left.\begin{array}{r} 3 \times [(\mathbb{1}, \mathbb{1}, \mathbf{8}) + (\mathbb{1}, \mathbb{1}, \mathbf{8}_s) + (\mathbb{1}, \mathbb{1}, \mathbf{8}_c)] \\ 9 \times \left[(\mathbf{6}, \mathbb{1}, \mathbb{1}) + (\overline{\mathbf{6}}, \mathbb{1}, \mathbb{1})\right] \\ 9 \times \left[(\mathbb{1}, \mathbf{3}, \mathbb{1}) + (\mathbb{1}, \overline{\mathbf{3}}, \mathbb{1})\right] \\ 18 \times (\mathbb{1}, \mathbb{1}, \mathbb{1}) \end{array}\right\} \text{twisted, no oscillators,}$$

$$\left.18 \times (\mathbb{1}, \mathbb{1}, \mathbb{1})\right\} \text{twisted, oscillators,}$$

$$\left.\begin{array}{r} (\mathbb{1}, \mathbb{1}, \mathbf{8}) + (\mathbb{1}, \mathbb{1}, \mathbf{8}_s) + (\mathbb{1}, \mathbb{1}, \mathbf{8}_c) \\ (\mathbf{20}, \mathbb{1}, \mathbb{1}) + 4 \times (\mathbb{1}, \mathbb{1}, \mathbb{1}) \end{array}\right\} \text{untwisted, no oscillators,}$$

$$\left.2 \times (\mathbb{1}, \mathbb{1}, \mathbb{1})\right\} \text{untwisted, oscillators.}$$

Matching moduli

- Crucial: **Higgsing** the orbifold gauge symmetry!

$$\mathrm{SU}(6) \times \underbrace{\mathrm{U}(1)^3}_{\text{3 hypers}} \times \left[\underbrace{\mathrm{SU}(3)}_{\text{8 hypers}} \times \mathrm{SO}(8) \times \underbrace{\mathrm{U}(1)^2}_{\text{2 hypers}}\right] \to \mathrm{SU}(6) \times \mathrm{SO}(8)$$

- Gauge bundle moduli, visible sector:

$$\underbrace{18}_{\substack{\text{twisted,}\\ \text{no osc.}}} + \underbrace{4}_{\substack{\text{untwisted,}\\ \text{no osc.}}} - 3 = 19$$

- Hidden sector:

$$\underbrace{54}_{\substack{\text{twisted,}\\ 9 \times [\mathbf{3} + \overline{\mathbf{3}}]}} - 10 = 44$$

- Geometrical moduli:

$$\underbrace{18}_{\substack{\text{twisted,}\\ \text{oscillators}}} + \underbrace{2}_{\substack{\text{untwisted,}\\ \text{oscillators}}} = 20$$

Conclusions

1. A 6D orbifold GUT with local $\mathrm{SU}(5)$ unification was derived from an anisotropic orbifold compactification.
2. Symmetry arguments can be used to select promising vacua, and to simplify the calculation of the superpotential.
3. Unbroken discrete symmetries can forbid disfavored terms to all orders, for example the μ-term.
4. After Higgsing, the 6D bulk spectrum can be matched with a compactification on $K3$.

Outlook

1. Compare $\frac{T^4/\mathbb{Z}_3 \times T^2}{\mathbb{Z}_2}$ and $\frac{K3 \times T^2}{\mathbb{Z}_2}$ compactifications.
2. Deduce the role of the fields. Understand the decoupling.
3. Find new guidelines towards the physical vacuum.

Backup slides

[Buchmüller, Lüdeling, JS 2007]

The effective orbifold GUT in 6D

$\mathrm{SU}(5)\times\mathrm{U}(1)^4\times\left[\mathrm{SU}(3)\times\mathrm{SO}(8)\times\mathrm{U}(1)^2\right]$

$(\mathbf{10},\mathbb{1},\mathbb{1})+(\overline{\mathbf{5}},\mathbb{1},\mathbb{1})+(\mathbb{1},\mathbb{1},\mathbb{1})$
$+2\times\left[(\mathbb{1},\mathbf{3},\mathbb{1})+(\mathbb{1},\overline{\mathbf{3}},\mathbb{1})\right]$
$+(\mathbb{1},\mathbb{1},\mathbf{8}_c)+8\cdot(\mathbb{1},\mathbb{1},\mathbb{1})$

$\mathrm{SU}(2)\times\mathrm{SU}(4)\times\mathrm{U}(1)^4\times\left[\mathrm{SU}(2)'\times\mathrm{SU}(4)'\times\mathrm{U}(1)^4\right]$

$4\cdot(\mathbf{2},\mathbb{1},\mathbb{1},\mathbb{1})+16\cdot(\mathbb{1},\mathbb{1},\mathbb{1},\mathbb{1})$

supergravity multiplet
tensor multiplet
2 geom. moduli hypermultiplets

$\mathrm{SU}(6)\times\mathrm{U}(1)^3\times\left[\mathrm{SU}(3)\times\mathrm{SO}(8)\times\mathrm{U}(1)^2\right]$

$9\times\left[(\mathbf{6},\mathbb{1},\mathbb{1})+(\overline{\mathbf{6}},\mathbb{1},\mathbb{1})\right]+(\mathbf{20},\mathbb{1},\mathbb{1})$
$+9\times\left[(\mathbb{1},\mathbf{3},\mathbb{1})+(\mathbb{1},\overline{\mathbf{3}},\mathbb{1})\right]$
$+4\times[(\mathbb{1},\mathbb{1},\mathbf{8})+(\mathbb{1},\mathbb{1},\mathbf{8}_c)+(\mathbb{1},\mathbb{1},\mathbf{8}_s)]$
$+40\times(\mathbb{1},\mathbb{1},\mathbb{1})$

$(\mathbf{10},\mathbb{1},\mathbb{1})+(\overline{\mathbf{5}},\mathbb{1},\mathbb{1})+(\mathbb{1},\mathbb{1},\mathbb{1})$
$+2\times\left[(\mathbb{1},\mathbf{3},\mathbb{1})+(\mathbb{1},\overline{\mathbf{3}},\mathbb{1})\right]$
$+(\mathbb{1},\mathbb{1},\mathbf{8}_c)+8\cdot(\mathbb{1},\mathbb{1},\mathbb{1})$

$\mathrm{SU}(5)\times\mathrm{U}(1)^4\times\left[\mathrm{SU}(3)\times\mathrm{SO}(8)\times\mathrm{U}(1)^2\right]$

$4\cdot(\mathbf{2},\mathbb{1},\mathbb{1},\mathbb{1})+16\cdot(\mathbb{1},\mathbb{1},\mathbb{1},\mathbb{1})$

$\mathrm{SU}(2)\times\mathrm{SU}(4)\times\mathrm{U}(1)^4\times\left[\mathrm{SU}(2)'\times\mathrm{SU}(4)'\times\mathrm{U}(1)^4\right]$

Localized Fayet–Iliopoulos *D*-terms

The model is a complicated interacting field theory. All anomalies

- either cancel among themselves,
- or by the variation of B_2 (Green–Schwarz).

[Buchmüller, Lüdeling, JS 2007]

There are **two localized anomalous U(1)'s**:

- GUT fixed points: $\operatorname{tr} t_{\mathrm{an}}^{0} = 1$
- Exotic fixed points: $\operatorname{tr} t_{\mathrm{an}}^{1} = \frac{1}{2}$

$$\left.\begin{array}{l} \operatorname{tr} t_{\mathrm{an}}^{0} = 1 \\ \operatorname{tr} t_{\mathrm{an}}^{1} = \frac{1}{2} \end{array}\right\} \quad t_{\mathrm{an}}^{\mathrm{4D}} = t_{\mathrm{an}}^{0} + \tfrac{1}{2} t_{\mathrm{an}}^{1}$$

Their generators are neither collinear nor orthogonal.

The 'false vacuum' (zero vevs) is not supersymmetric:

$$D = F_{56} - \xi - \sum_i q_i |\phi_i|^2, \qquad \xi = \frac{g M_P^2}{384\pi^2} \operatorname{tr} t \sim M_{\mathrm{GUT}}^2$$

Orbifolds generically require large vevs. Consistent?