Intersecting branes on smooth Calabi-Yau manifolds

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Motivations

- Setting: Global models of intersecting D6 branes in IIA
- Study supersymmetric intersecting branes on non-torodial CYs
- Unexplored branch of the landscap (although see Gepner models)
- Advance connection with moduli stabilisation on CYs

Background

- Study of supersymmetric D-branes on the Fermat quintic $\psi = 0$

Brunner et al. '00

$$P = z_1^5 + z_2^5 + z_3^5 + z_4^5 + z_5^5 + \psi z_1 z_2 z_3 z_4 z_5 = 0 \qquad z_i \in CP^4$$

$\longrightarrow$ Intersection matrix of a set of 625 special Lagrangian submanifolds

- Search for appropriate intersections to give (non-susy) SM ✓

Blumenhagen et al. '02

- No supersymmetric chiral models (stability?)
- What about other manifolds?

Hypersurfaces in weighted projective spaces

• Simple class of CY manifolds are given by hypersurfaces (or intersections of) in (possibly weighted) projective spaces

$$CP^4_{[5,2,1,1,1]}$$

$$P = z_1^2 + z_2^5 + z_3^{10} + z_4^{10} + z_5^{10} + \psi z_1 z_2 z_3 z_4 z_5 = 0$$

• There are 7890 CICYs and 7555 hypersurfaces in weighted projective spaces.

• Require smooth and concentrate on weighted projective spaces

$$CP^4_{[5,2,1,1,1]} \qquad CP^4_{[4,1,1,1,1]} \qquad CP^4_{[2,1,1,1,1]}$$

• Depending on moduli there are large discrete symmetry groups associated to rotations of the co-ordinates.

$$CP^4_{[5,2,1,1,1]} \qquad \frac{\mathbb{Z}_2 \times \mathbb{Z}_5 \times \mathbb{Z}_{10} \times \mathbb{Z}_{10} \times \mathbb{Z}_{10}}{\mathbb{Z}_{10} \times \mathbb{Z}_{10}}$$

Special Lagrangians from involutions

- Special Lagrangian submanifold Π is defined by

$$\mathrm{Im}\left(e^{i\frac{\theta_\Pi}{2}}\Omega\right)\Big|_\Pi = 0\ ,\ \ J|_\Pi = 0\ , \qquad\qquad \epsilon_\Pi = \mathrm{Re}\left(e^{i\frac{\theta_\Pi}{2}}\Omega\right)\Big|_\Pi\ .$$

- Isometric anti-holomorphic involution

$$\sigma(J) = -J\ ,\ \ \sigma(\Omega) = \overline{\left(e^{i\theta}\Omega\right)}$$

Concentrate on type:

$$\sigma(z_i) = \overline{(\omega_i z_i)} \qquad\qquad P_A(\omega_i z_i) = P_A(z_i)$$

- Fixed point locus is a special Lagrangian manifold

$$P_A(\mathrm{Re}\,(\omega_i^{\frac{1}{2}} z_i)) \ = \ 0 \qquad\qquad \mathrm{Im}\,(\omega_i^{\frac{1}{2}} z_i) \ = \ 0$$

- Supersymmetry given by calibration angles $\quad e^{i\theta_\sigma} = \prod_i \omega_i$

Special Lagrangians from involutions

- So a special Lagrangian is specified by a set of rotation angles and an orientation

$$\Pi = \{\omega_i\}_p$$

- Actually (like orbifolds) need to sum over related angles

$$CP^n_{[w_i]} = \frac{CP^n}{\Pi_i \, \mathbb{Z}_{w_i}} \,. \qquad \Pi^{P_{j_p}} = \sum_{k_{j_p}} \left\{ \left(\alpha_{j_p}\right)^{k_{j_p} w_i} \omega_i \right\}_{p(k_{j_p})}$$

- Here the superscript P_{j_p} denotes a local patch given by setting z_{j_p} to one and α denotes the w_{j_p} root of unity.

Intersection numbers

- The special Lagrangian cycles can intersect at points or surfaces.
- Only point intersections can be supersymmetric.
- Point intersections are simply calculated by the number of point solutions to the two defining equations.
- Must sum over the patches on the manifold – can give multiple intersections.
- Rank of intersection matrix gives the homology span

Ambient Space	Defining Polynomial	SLAG	SUSY	b^3	Rank
$CP^4_{[1,1,1,1,1]}$	$P = \eta_1^0 z_1^5 + \eta_2^0 z_2^5 + \eta_3^0 z_3^5 + \eta_4^0 z_4^5 + \eta_5^0 z_5^5 = 0$	625	125	204	204
$CP^4_{[2,1,1,1,1]}$	$P = \eta_1^0 z_1^3 + \eta_2^0 z_2^6 + \eta_3^0 z_3^6 + \eta_4^0 z_4^6 + \eta_5^0 z_5^6 = 0$	648	108	208	54
$CP^4_{[4,1,1,1,1]}$	$P = \eta_1^0 z_1^2 + \eta_2^0 z_2^8 + \eta_3^0 z_3^8 + \eta_4^0 z_4^8 + \eta_5^0 z_5^8 = 0$	960	120	300	82
$CP^4_{[5,2,1,1,1]}$	$P = \eta_1^0 z_1^2 + \eta_2^0 z_2^5 + \eta_3^0 z_3^{10} + \eta_4^0 z_4^{10} + \eta_5^0 z_5^{10} = 0$	1000	100	292	100

Intersecting Branes

- Wrap orientifold on a chosen special Lagrangian.
- Supersymmetry requires brane cycles share calibration.
- Tadpole constraint reads

$$\sum_a N_a \left(\Pi_a + \Pi_{a'}\right) - 4\Pi_{O6} = 0 .$$

- Since only know intersection numbers can rewrite as

$$\sum_a N_a \left(I_{ab} + I_{a'b}\right) - 4I_{O6,b} = 0 \qquad \forall\ b$$

- The two are equivalent if we intersect with a set that spans the full homology of the manifold.

- Otherwise can require the weaker condition of intersecting with only visible gauge group branes: anomaly cancellation and no new exotics but need global completion.

- Chiral spectrum given as usual by the intersection numbers $I_{ab} = (n_a, \bar{n}_b)$

Model Building

• Computer model search on weighted projective spaces and a selection of CICYs.

• No GUT models: could not find three copies of the anti-symmetric representation of SU(5).

• Pati-Salam model on $CP^4_{[5,2,1,1,1]}$

$$\Pi_a = \{0,0,0,3,7\}_- \quad , \quad \Pi_{a'} = \{0,0,0,7,3\}_-$$
$$\Pi_b = \{0,0,7,2,1\}_- \quad , \quad \Pi_{b'} = \{0,0,3,8,9\}_+$$
$$\Pi_c = \{0,0,3,9,8\}_+ \quad , \quad \Pi_{c'} = \{0,0,7,1,2\}_-$$

	Π_a	Π_b	Π_c	$\Pi_{a'}$	$\Pi_{b'}$	$\Pi_{c'}$	Π_0
Π_a	0	-1	1	0	-2	2	0
Π_b		0	1		3	0	1
Π_c			0			-3	-1

Field	Multiplicity	Representation
Q_L	3	$(4,2,1)$
Q_R	3	$(\bar{4},1,2)$
h	1	$(1,2,2)$
H_+	1	$(\bar{4},1,2)$
H_-	1	$(4,1,2)$
B_1	1	$[S]_{SU(2)}$
B_2	2	$[A]_{SU(2)}$
C_1	1	$[S]_{SU(2)}$
C_2	2	$[A]_{SU(2)}$

Model Building

- MSSM-like model on $CP^4_{[5,2,1,1,1]}$

$$\Pi_a = \{0,0,0,3,7\}_- \;,\; \Pi_{a'} = \{0,0,0,7,3\}_- \;,$$
$$\Pi_b = \{0,0,3,8,9\}_+ \;,\; \Pi_{b'} = \{0,0,7,2,1\}_- \;,$$
$$\Pi_c = \{0,0,3,0,7\}_- \;,\; \Pi_{c'} = \{0,0,7,0,3\}_- \;,$$
$$\Pi_d = \{0,0,4,1,5\}_- \;,\; \Pi_{d'} = \{0,0,6,9,5\}_+ \;,$$
$$\Pi_e = \{0,0,7,8,5\}_- \;,\; \Pi_{e'} = \{0,0,3,2,5\}_+ \;,$$
$$\Pi_f = \{0,0,2,6,2\}_- \;,\; \Pi_{f'} = \{0,0,8,4,8\}_+ \;,$$
$$\Pi_g = \{0,0,3,4,3\}_- \;,\; \Pi_{g'} = \{0,0,7,6,7\}_+ \;.$$

Field	Multiplicity	Representation
Q	3	$(3,2)_{\frac{1}{6}}$
U	3	$(\bar{3},1)_{-\frac{2}{3}}$
D	3	$(\bar{3},1)_{\frac{1}{3}}$
L	3	$(1,2)_{-\frac{1}{2}}$
E	3	$(1,1)_1$
N	3	$(1,1)_0$
$\mathrm{H_u}$	1	$(1,2)_{\frac{1}{2}}$
$\mathrm{H_d}$	1	$(1,2)_{-\frac{1}{2}}$
$\mathrm{H_1}$	1	$(1,2)_{\frac{1}{2}}$
$\mathrm{H_2}$	1	$(1,2)_{-\frac{1}{2}}$
$\mathrm{B_1}$	1	$(\bar{3},1)_{-\frac{2}{3}}$
$\mathrm{B_2}$	1	$(\bar{3},1)_{\frac{2}{3}}$
$\mathrm{B_3}$	1	$(\bar{3},1)_{-\frac{1}{2}}$
$\mathrm{B_4}$	1	$(3,1)_{\frac{1}{2}}$
$\mathrm{C_1}$	4	$(1,2)_0$
$\mathrm{D_1}$	6	$(1,1)_{\frac{1}{2}}$
$\mathrm{D_2}$	7	$(1,1)_{-\frac{1}{2}}$
$\mathrm{E_1}$	1	$[\mathrm{S}]_{SU(2)}$
$\mathrm{F_1}$	3	$(1,1)_X$
$\mathrm{F_2}$	3	$(1,1)_X$

Summary

- Studied intersecting D6 branes on smooth non-torodial CYs

- Found supersymmetric chiral (semi-realistic) models.

- In the cases studied the set of special Lagrangians did not span the full homology of the manifold implying the models are incomplete.

- Can enlarge the special Lagrangian set by studying fixed points of anti-holomorphic permutations.

- Can study singular hypersurfaces that are blown up to CYs. This would greatly increase the possible models.

- Models rely on locus in moduli space – need to study interaction with moduli stabilisation.

Intersecting Branes: U(1)s

• To determine if a U(1) remains massless need to span the full homology

$$U(1) = \sum_a Q_a U(1)_a \ , \quad \text{massless if} \quad \sum_a N_a Q_a \left(\Pi_a - \Pi_{a'}\right) = 0 \ .$$

• Can also consider the weaker condition

$$\sum_a N_a Q_a \left(I_{ab} - I_{a'b}\right) = 0 \ , \ \ \forall \, b$$

This would imply that if we could add a brane to make the U(1) massless it would not give rise to new chiral charged exotics.