Effective equations of motion on the brane in higher order dilaton gravity

Dominika Konikowska

University of Warsaw,
Institute of Theoretical Physics

String Pheno - Warsaw, 18 June 2009

Outline

what if ‘some’ extra dimensions existed . . .

- currently observed 4 space-time dimensions
  - Einstein equations of motion
  - Einstein-Hilbert action

  $\longrightarrow$ linear in Riemann tensor
- higher-dimensional space-times

  $\rightsquigarrow$ additional higher curvature terms can be considered
- introducing higher powers of Riemann tensor into the gravity action

  $\rightsquigarrow$ Einstein theory of gravity generalized
- for a given order in the Riemann tensor

  $\longrightarrow$ contribution to the action unique (overall normalization)
  - quadratic contribution: Gauss-Bonnet (Lanczos) term
  - generalized to higher orders by Lovelock

what if ‘some’ extra dimensions existed ...

- currently observed 4 space-time dimensions
  - Einstein equations of motion
  - Einstein-Hilbert action

    $\longrightarrow$ linear in Riemann tensor

- higher-dimensional space-times

  $\leadsto$ additional higher curvature terms can be considered
- introducing higher powers of Riemann tensor into the gravity action

  $\leadsto$ Einstein theory of gravity generalized
- for a given order in the Riemann tensor

  $\longrightarrow$ contribution to the action unique (overall normalization)
  - quadratic contribution: Gauss-Bonnet (Lanczos) term
  - generalized to higher orders by Lovelock

string theory behind the scenes?

- effective action obtained from string theories
  $\rightsquigarrow$ higher derivative corrections to the gravity interactions
- first correction exactly of the form of the Gauss-Bonnet term
  $\pm$ local field redefinitions
- *dilaton*: scalar field of the gravitational sector
- α' expansion in the string theories
  $\rightsquigarrow$ higher order corrections also for the dilaton interactions

string theory behind the scenes?

- effective action obtained from string theories

  $\rightsquigarrow$ higher derivative corrections to the gravity interactions
- first correction exactly of the form of the Gauss-Bonnet term

  $\pm$ local field redefinitions

- *dilaton*: scalar field of the gravitational sector
- α' expansion in the string theories

  $\rightsquigarrow$ higher order corrections also for the dilaton interactions

Outline

construction of the dilaton gravity equations of motion

- Einstein-Lovelock higher order gravity

  $\rightsquigarrow$ couple to the dilaton

- *N*-th order dilaton gravity equations of motion: $T^{(N)}_{\mu\nu} = 0$ & $W^{(N)} = 0$

  $\rightsquigarrow$ *constructed*

  - at each order: unique up to a normalization

- *E-L theory generalized* $\rightarrow$ higher order dilaton gravity

some useful notation

- a generalization of the Kronecker delta

$$\delta^{\sigma_1\sigma_2\cdots\sigma_N}_{\rho_1\rho_2\cdots\rho_N} = \det\begin{vmatrix} \delta^{\sigma_1}_{\rho_1} & \delta^{\sigma_1}_{\rho_2} & \cdots & \delta^{\sigma_1}_{\rho_N} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \delta^{\sigma_N}_{\rho_1} & \delta^{\sigma_N}_{\rho_2} & \cdots & \delta^{\sigma_N}_{\rho_N} \end{vmatrix}$$

- a generalization of the trace operator

$$\mathcal{T}(M) = \delta^{\sigma_1\sigma_2\cdots\sigma_N}_{\rho_1\rho_2\cdots\rho_N} M^{\rho_1\rho_2\cdots\rho_N}{}_{\sigma_1\sigma_2\cdots\sigma_N}$$

- an extension of the trace operator

$$\overline{\mathcal{T}}{}^{\nu}_{\mu}(M) = \delta^{\nu\,\sigma_1\sigma_2\cdots\sigma_N}_{\mu\,\rho_1\rho_2\cdots\rho_N} M^{\rho_1\rho_2\cdots\rho_N}{}_{\sigma_1\sigma_2\cdots\sigma_N}$$

some useful notation → example

- and a generalization of the *N*-th power operator

$$\mathcal{T}\left(\left[\tfrac{1}{2}\mathcal{R}^{**}_{**} \oplus 2(\nabla\nabla)^{*}_{*}\phi\right]^{2}\right) =$$

$$= \tfrac{1}{4}\,\mathcal{T}(\mathcal{R}^{**}_{**}\,\mathcal{R}^{**}_{**}) + 2\,\mathcal{T}(\mathcal{R}^{**}_{**}\,(\nabla\nabla)^{*}_{*}\phi) + 4\,\mathcal{T}((\nabla\nabla)^{*}_{*}\phi\,(\nabla\nabla)^{*}_{*}\phi) =$$

$$= \tfrac{1}{4}\,\delta^{\sigma_1\sigma_2\sigma_3\sigma_4}_{\rho_1\rho_2\rho_3\rho_4}\,\mathcal{R}^{\rho_1\rho_2}{}_{\sigma_1\sigma_2}\,\mathcal{R}^{\rho_3\rho_3}{}_{\sigma_3\sigma_4} +$$

$$+ 2\,\delta^{\sigma_1\sigma_2\sigma_3}_{\rho_1\rho_2\rho_3}\,\mathcal{R}^{\rho_1\rho_2}{}_{\sigma_1\sigma_2}\left(\nabla^{\rho_3}\partial_{\sigma_3}\phi\right) +$$

$$+ 4\,\delta^{\sigma_1\sigma_2}_{\rho_1\rho_2}\left(\nabla^{\rho_1}\partial_{\sigma_1}\phi\right)\left(\nabla^{\rho_2}\partial_{\sigma_2}\phi\right)$$

- asterisks → tensors ranks
- shorthand: $(\nabla\nabla)^{\rho}_{\sigma} \equiv \nabla^{\rho}\partial_{\sigma}$

(just acquired) starting point: *d*-dimensional higher order dilaton gravity

- *d*-dimensional tensor $T_{\mu\nu} = 0$ and scalar $W = 0$ equations of motion

$$-\sum_{N=1}^{N_{max}} \frac{\alpha_N}{2} \overline{\mathcal{T}}_{\mu\nu}\left(\left[\tfrac{1}{2}\mathcal{R}^{**}_{**} \oplus 2(\nabla\nabla)^*_*\phi \oplus (-1)(\partial\phi)^2\right]^N\right) + g_{\mu\nu}V(\phi) - \tau_{\mu\nu}\delta_B = 0$$

$$-\sum_{N=1}^{N_{max}} \frac{\alpha_N}{2} \mathcal{T}\left(\left[\tfrac{1}{2}\mathcal{R}^{**}_{**} \oplus 2(\nabla\nabla)^*_*\phi \oplus (-1)(\partial\phi)^2\right]^N\right) + V(\phi) - V'(\phi) - \tau_\phi\delta_B = 0$$

  - position of the brane: Dirac delta type distribution δ_B
  - brane localized terms: $\tau_{\mu\nu} = h_{\mu\nu}\mathcal{L}_B - 2\frac{\delta\mathcal{L}_B}{\delta h^{\mu\nu}}$ & $\tau_\phi = \mathcal{L}_B - \frac{\delta\mathcal{L}_B}{\delta\phi}$ due to the brane interactions given by $\mathcal{L}_B$
  - induced brane metric: $h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$
    - n^μ: unit vector field normal to the brane (at the brane)

- corresponding Lagrangian density

$$\mathcal{L} = e^{-\phi}\left\{ -V(\phi) + \mathcal{L}_B\delta_B + \sum_{N=1}^{N_{max}} \frac{\alpha_N}{2} \mathcal{T}\left(\left[\tfrac{1}{2}\mathcal{R}^{**}_{**} \oplus 2(\nabla\nabla)^*_*\phi \oplus (-1)(\partial\phi)^2\right]^N\right)\right\}$$

Outline

- introduction
- higher order dilaton gravity
- effective brane equations
- cosmological example
- higher order & conclusions

where to?

- brane-world ideology: standard model localized on a brane

  $\longrightarrow$ embedded in higher dimensional space-time

  $\rightsquigarrow$ how will the induced gravity look like on the brane?

- effective equations of motion: $(d-1)$-dimensional

  $\rightsquigarrow$ simply restricting full d-dimensional equations? NO!

- certain quantities contributing to $T^{(N)}_{\mu\nu} = 0$ and $W^{(N)} = 0 \ldots$ can be singular or discontinuous on the brane
  - singular: explicit Dirac delta contributions or discontinuous functions derivatives

  $\rightsquigarrow$ *non-trivial derivation* of effective equations on the brane

  $\rightsquigarrow$ will be carried out in COVARIANT APPROACH

where to?

- brane-world ideology: standard model localized on a brane

  $\longrightarrow$ embedded in higher dimensional space-time

  $\rightsquigarrow$ how will the induced gravity look like on the brane?

- effective equations of motion: $(d-1)$-dimensional

  $\rightsquigarrow$ simply restricting full d-dimensional equations? NO!

- certain quantities contributing to $T^{(N)}_{\mu\nu} = 0$ and $W^{(N)} = 0 \ldots$ can be singular or discontinuous on the brane

  - singular: explicit Dirac delta contributions or discontinuous functions derivatives

  $\rightsquigarrow$ *non-trivial derivation* of effective equations on the brane

  $\rightsquigarrow$ will be carried out in COVARIANT APPROACH

where to?

- brane-world ideology: standard model localized on a brane

  $\longrightarrow$ embedded in higher dimensional space-time

  $\rightsquigarrow$ how will the induced gravity look like on the brane?

- effective equations of motion: $(d-1)$-dimensional

  $\rightsquigarrow$ simply restricting full d-dimensional equations? NO!

- certain quantities contributing to $T^{(N)}_{\mu\nu} = 0$ and $W^{(N)} = 0 \ldots$ can be singular or discontinuous on the brane
  - singular: explicit Dirac delta contributions or discontinuous functions derivatives

  $\rightsquigarrow$ *non-trivial derivation* of effective equations on the brane

  $\rightsquigarrow$ will be carried out in COVARIANT APPROACH

projecting: “parallel” & “perpendicular” to the brane

- identification in relevant tensors: benefits from $\mathcal{T}\&\overline{\mathcal{T}}$ antisymmetrization

$$\mathcal{R}^{**}_{**} \rightarrow R^{**}_{**} - 2K^*_*K^*_* - 4(nn)^*_*\{\pounds_n K^*_* - (KK)^*_*\} - 8(nD)^*_*K^*_*$$

$$(\nabla\nabla)^*_*\phi \rightarrow \left[(DD)^*_*\phi + K^*_*\pounds_n\phi\right] + (nn)^*_*\left\{\pounds_n{}^2\phi - a^e\nabla_e\phi\right\} + $$

$$+2\left[(nD)^*_*\pounds_n\phi - (nKD)^*_*\phi\right]$$

$$(\partial\phi)^2 = (D\phi)^2 + (\pounds_n\phi)^2$$

  - $g_{\mu\nu}$: $\mathcal{R}^{\mu\nu}_{\rho\sigma}$ & ∇_μ vs $h_{\mu\nu}$: $R^{\mu\nu}_{\rho\sigma}$ & D_μ
  - $K_{\mu\nu}$: extrinsic curvature of hypersurfaces orthogonal to n^μ
  - $\pounds_n$: Lie derivative along n^μ
  - $a^e\nabla_e\phi = n^a(\nabla_a n^b)(\nabla_b\phi)$ (‘non-typical’; not present in final results)
  - shorthand again: $(nn)^*_* \equiv n_* n^*$, $(DD)^*_* \equiv D_* D^*$, $(KK)^*_* \equiv K^*_x K^x_*$, $(nD)^*_* \equiv \frac{1}{2}(n_* D^* + n^* D_*)$, $(nKD)^*_* \equiv \frac{1}{2}(n_* K^*_x D^x + n^* K^x_* D_x)$

effective gravitational equations on the brane . . . identifying problems

- the good

  $\longrightarrow \quad h_{\mu\nu}$, $R_{\mu\nu}$, $(DD)_{\mu\nu}\phi$, $(D\phi)^2$, $V(\phi)$

  $\rightsquigarrow$ no work needed here, rejoice!

- the kind of bad

  $\longrightarrow \quad K_{\mu\nu}$, $\pounds_n\phi$

  $\rightsquigarrow$ can be discontinuous when 'crossing' the brane

- and the slightly ugly

  $\longrightarrow \quad \pounds_n K_{\mu\nu}$, ${\pounds_n}^2\phi$

  $\rightsquigarrow$ can be singular on the brane

  $\rightsquigarrow$ can have a finite contribution as well

$\rightsquigarrow$ all this information has to be properly taken into account

& the quest begins

effective gravitational equations on the brane . . . identifying problems

- the good

  → $h_{\mu\nu}$, $R_{\mu\nu}$, $(DD)_{\mu\nu}\phi$, $(D\phi)^2$, $V(\phi)$

  ⇝ no work needed here, rejoice!

- the kind of bad

  → $K_{\mu\nu}$, $\pounds_n\phi$

  ⇝ can be discontinuous when 'crossing' the brane

- and the slightly ugly

  → $\pounds_n K_{\mu\nu}$, ${\pounds_n}^2\phi$

  ⇝ can be singular on the brane

  ⇝ can have a finite contribution as well

⇝ all this information has to be properly taken into account

& the quest begins

effective gravitational equations on the brane … identifying problems

- the good

  $\longrightarrow \quad h_{\mu\nu}, R_{\mu\nu}, (DD)_{\mu\nu}\phi, (D\phi)^2, V(\phi)$

  $\rightsquigarrow$ no work needed here, rejoice!

- the kind of bad

  $\longrightarrow \quad K_{\mu\nu}, \pounds_n\phi$

  $\rightsquigarrow$ can be discontinuous when ‘crossing’ the brane

- and the slightly ugly

  $\longrightarrow \quad \pounds_n K_{\mu\nu}, {\pounds_n}^2\phi$

  $\rightsquigarrow$ can be singular on the brane

  $\rightsquigarrow$ can have a finite contribution as well

$\rightsquigarrow$ all this information has to be properly taken into account

& the quest begins

effective gravitational equations on the brane . . . identifying problems

- the good

  $\longrightarrow \quad h_{\mu\nu}, R_{\mu\nu}, (DD)_{\mu\nu}\phi, (D\phi)^2, V(\phi)$

  $\rightsquigarrow$ no work needed here, rejoice!

- the kind of bad

  $\longrightarrow \quad K_{\mu\nu}, \pounds_n\phi$

  $\rightsquigarrow$ can be discontinuous when 'crossing' the brane

- and the slightly ugly

  $\longrightarrow \quad \pounds_n K_{\mu\nu}, {\pounds_n}^2\phi$

  $\rightsquigarrow$ can be singular on the brane

  $\rightsquigarrow$ can have a finite contribution as well

$\rightsquigarrow$ all this information has to be properly taken into account

& the quest begins

effective gravitational equations on the brane . . . addressing problems

- terms discontinuous on the brane (leading to singularities)

  $\longrightarrow K_{\mu\nu}, \pounds_n\phi$

  $\rightsquigarrow$ *junction* (boundary) *conditions*

- terms singular on the brane

  $\longrightarrow \pounds_n K_{\mu\nu}, {\pounds_n}^2\phi$

  - purely singular contributions already addressed by the junction conditions
  - the smooth part has to be determined as well
    (yields a finite contribution to the effective equations)

  $\rightsquigarrow$ *"brane limit of bulk equations system"*

  - take scalar equation of motion & the trace of tensor equation of motion
    $\longrightarrow$ "bulk equations system"
  - now the brane limit (i.e. evaluate 'next to the brane')
  - solve it

effective gravitational equations on the brane ... addressing problems

- terms discontinuous on the brane (leading to singularities)

  $\longrightarrow K_{\mu\nu}, \pounds_n \phi$

  $\rightsquigarrow$ *junction* (boundary) *conditions*

- terms singular on the brane

  $\longrightarrow \pounds_n K_{\mu\nu}, {\pounds_n}^2 \phi$

  - purely singular contributions already addressed by the junction conditions
  - the smooth part has to be determined as well
    (yields a finite contribution to the effective equations)

  $\rightsquigarrow$ *"brane limit of bulk equations system"*

  - take scalar equation of motion & the trace of tensor equation of motion
    $\longrightarrow$ "bulk equations system"
  - now the brane limit (i.e. evaluate 'next to the brane')
  - solve it

effective gravitational equations on the brane . . .

- any order? complexity rather overwhelming
  $\rightsquigarrow$ $N = 1$ & $d = 5$ example
  - *first order dilaton gravity*
  - and a 4-dimensional brane
- higher order dilaton gravity: exactly the same procedure
  $\rightsquigarrow$ appropriate formulae derived just as well
  - explicit results? even fancy notation not always sufficient

effective gravitational equations . . . bulk equations projected ($N = 1$)

- “parallel-parallel” part $T^{\|\|}_{\mu\nu} = 0$

$$\left[\left\{ R_{\mu\nu} + (DD)_{\mu\nu}\phi - \frac{h_{\mu\nu}}{2}\left(R + 2(DD)\phi - (D\phi)^2\right) + h_{\mu\nu}\frac{V(\phi)}{\alpha_1} \right\} + \right.$$

$$+ \left\{ \left((KK)_{\mu\nu} - K_{\mu\nu}\left(K - \pounds_n\phi\right)\right) - \frac{h_{\mu\nu}}{2}\left((KK) - (K - \pounds_n\phi)^2\right) \right\} +$$

$$\left. - \left\{ \left(\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\right) - h_{\mu\nu}\left((h\pounds_n K) - (KK)\right) - h_{\mu\nu}\left({\pounds_n}^2\phi - a^e\nabla_e\phi\right) \right\} \right]_{\pm} = 0$$

addressing the discontinuities' problem: junction conditions

- junction conditions for a given point x_0^μ on the brane:

  $\longrightarrow$ integrating the d-dimensional equations of motion 'across-the-brane'

  i.e. in the direction perpendicular to the brane

  $\longrightarrow$ and shrinking the interval: 'infinitesimal across-the-brane integration'

- only some terms in the equations of motions $\nrightarrow$ zero
  - explicit brane contributions proportional to δ_B
  - terms containing second Lie derivatives: ${\pounds_n}^2\phi$ and $\pounds_n K_{\mu\nu}$

- useful notation
  - discontinuous at the brane $\rightsquigarrow$ jump: $[f(x_0)]_\pm = [f(x_0)]_+ - [f(x_0)]_-$
  - "brane limits": $[f(x_0)]_+$, $[f(x_0)]_-$

addressing the discontinuities' problem: junction conditions

- junction conditions for a given point x_0^μ on the brane:

  $\longrightarrow$ integrating the d-dimensional equations of motion 'across-the-brane'

  i.e. in the direction perpendicular to the brane

  $\longrightarrow$ and shrinking the interval: 'infinitesimal across-the-brane integration'
- only some terms in the equations of motions $\not\rightarrow$ zero
  - explicit brane contributions proportional to δ_B
  - terms containing second Lie derivatives: $\pounds_n{}^2\phi$ and $\pounds_n K_{\mu\nu}$
- useful notation
  - discontinuous at the brane $\rightsquigarrow$ jump: $[f(x_0)]_\pm = [f(x_0)]_+ - [f(x_0)]_-$
  - "brane limits": $[f(x_0)]_+$, $[f(x_0)]_-$

addressing the discontinuities' problem: junction conditions

- junction conditions for a given point x_0^μ on the brane:

  $\longrightarrow$ integrating the d-dimensional equations of motion 'across-the-brane'

  i.e. in the direction perpendicular to the brane

  $\longrightarrow$ and shrinking the interval: 'infinitesimal across-the-brane integration'

- only some terms in the equations of motions $\nrightarrow$ zero
  - explicit brane contributions proportional to δ_B
  - terms containing second Lie derivatives: $\pounds_n{}^2\phi$ and $\pounds_n K_{\mu\nu}$

- useful notation
  - discontinuous at the brane $\rightsquigarrow$ jump: $[f(x_0)]_\pm = [f(x_0)]_+ - [f(x_0)]_-$
  - "brane limits": $[f(x_0)]_+$, $[f(x_0)]_-$

junction conditions: $N = 1 \rightsquigarrow [K_{\mu\nu}]_\pm$ & $[\pounds_n \phi]_\pm$ explicitly

- tensor junction condition (effectively from $T^{|||}_{\mu\nu} = 0$)
- scalar junction condition

$\rightsquigarrow$ easily solvable, jumps can be determined

$$\left[K_{\mu\nu}\right]_\pm = \frac{1}{\alpha_1}\left(h_{\mu\nu}\tau_\phi - \tau_{\mu\nu}\right)$$
$$\left[\pounds_n \phi\right]_\pm = \frac{1}{\alpha_1}\left((d-2)\tau_\phi - \tau\right)$$

- however, no information whatsoever about the brane limits

  $\rightsquigarrow$ unless …

junction conditions: $N = 1$ & $\mathbb{Z}_2$ $\rightsquigarrow$ $[K_{\mu\nu}]_+$ & $[\pounds_n\phi]_+$ explicitly

- $\mathbb{Z}_2$ symmetry, brane located at the orbifold fixed point

  $\rightsquigarrow$ $[f]_\pm = 2[f]_+$ if f is $\mathbb{Z}_2$-odd, i.e. $[f]_- = -[f]_+$

$\rightsquigarrow$ brane limits can be determined

$$[K_{\mu\nu}]_+ = \frac{1}{2\alpha_1}\left(h_{\mu\nu}\tau_\phi - \tau_{\mu\nu}\right)$$

$$[\pounds_n\phi]_+ = \frac{1}{2\alpha_1}\left((d-2)\tau_\phi - \tau\right)$$

addressing the discontinuities' problem: bulk equations system

- terms singular on the brane

  $\rightarrow$ tend to appear in equations of motion in certain combinations

  $\rightarrow \{\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\}$, $\{\pounds_n{}^2\phi - a^e\nabla_e\phi\}$

  $\rightsquigarrow$ good occasion to get rid of $a^e\nabla_e\phi$ as well

- "brane limit of bulk equations system"
  - supposed to yield finite contributions to $\pounds_n K_{\mu\nu}$ & $\pounds_n{}^2\phi$
  - a system of scalar equations ($W^{(N)} = 0$ and trace of $T^{(N)}_{\mu\nu} = 0$)
  - but $\pounds_n K_{\mu\nu}$ is a tensor variable... how come it can work?

    $\rightarrow$ yes, it can

$$\left\{\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\right\} = \frac{h_{\mu\nu}}{d-1}\left\{(h\pounds_n K) - (KK)\right\} - \frac{1}{d-3}\left(R_{\mu\nu} - KK_{\mu\nu} + (KK)_{\mu\nu}\right) + \frac{h_{\mu\nu}}{(d-1)(d-3)}\left(R - K^2 + (KK)\right) - \frac{d-2}{d-3}E_{\mu\nu}$$

addressing the discontinuities' problem: bulk equations system

- terms singular on the brane

  $\longrightarrow$ tend to appear in equations of motion in certain combinations

  $\longrightarrow \{\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\}$, $\{\pounds_n{}^2\phi - a^e\nabla_e\phi\}$

  $\rightsquigarrow$ good occasion to get rid of $a^e\nabla_e\phi$ as well
- "brane limit of bulk equations system"
  - supposed to yield finite contributions to $\pounds_n K_{\mu\nu}$ & $\pounds_n{}^2\phi$
  - a system of scalar equations ($W^{(N)} = 0$ and trace of $T^{(N)}_{\mu\nu} = 0$)
  - but $\pounds_n K_{\mu\nu}$ is a tensor variable... how come it can work?

  $\longrightarrow$ yes, it can

$$\left\{\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\right\} = \frac{h_{\mu\nu}}{d-1}\left\{(h\pounds_n K) - (KK)\right\} - \frac{1}{d-3}\left(R_{\mu\nu} - KK_{\mu\nu} + (KK)_{\mu\nu}\right) + \\ + \frac{h_{\mu\nu}}{(d-1)(d-3)}\left(R - K^2 + (KK)\right) - \frac{d-2}{d-3}E_{\mu\nu}$$

addressing the discontinuities' problem: bulk equations system

- terms singular on the brane

  $\longrightarrow$ tend to appear in equations of motion in certain combinations

  $\longrightarrow \{\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\}$, $\{\pounds_n{}^2\phi - a^e\nabla_e\phi\}$

  $\rightsquigarrow$ good occasion to get rid of $a^e\nabla_e\phi$ as well

- "brane limit of bulk equations system"
  - supposed to yield finite contributions to $\pounds_n K_{\mu\nu}$ & $\pounds_n{}^2\phi$
  - a system of scalar equations ($W^{(N)} = 0$ and trace of $T^{(N)}_{\mu\nu} = 0$)
  - but $\pounds_n K_{\mu\nu}$ is a tensor variable... how come it can work?

    $\longrightarrow$ yes, it can

$$\left\{\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\right\} = \frac{h_{\mu\nu}}{d-1}\left\{(h\pounds_n K) - (KK)\right\} - \frac{1}{d-3}\left(R_{\mu\nu} - KK_{\mu\nu} + (KK)_{\mu\nu}\right) + \\ + \frac{h_{\mu\nu}}{(d-1)(d-3)}\left(R - K^2 + (KK)\right) - \frac{d-2}{d-3}E_{\mu\nu}$$

addressing the discontinuities' problem: bulk equations system

- $E_{\mu\nu} \equiv C_{abcd} h^a_\mu n^b h^c_\nu n^d$, where C_{abcd}: bulk Weyl tensor

  $\rightsquigarrow$ $E_{\mu\nu}$ enters $T^{||||}_{\mu\nu} = 0$

  (so promising as effective gravitational equations on the brane)

  $\longrightarrow$ to *never leave* it

- treating $T^{||||}_{\mu\nu} = 0$ as effective gravitational equations on the brane ...
  - single bulk associated variable $E_{\mu\nu}$

    $\rightsquigarrow$ describes the permanent influence of bulk teory on brane-world gravity
  - *not a closed system*

    $\rightsquigarrow$ bulk solutions essential to fully describe the gravity induced on the brane

addressing the discontinuities' problem: bulk equations system

- $E_{\mu\nu} \equiv C_{abcd} h^a_\mu n^b h^c_\nu n^d$, where C_{abcd}: bulk Weyl tensor

  $\rightsquigarrow$ $E_{\mu\nu}$ enters $T^{||||}_{\mu\nu} = 0$

  (so promising as effective gravitational equations on the brane)

  $\longrightarrow$ to *never leave* it

- treating $T^{||||}_{\mu\nu} = 0$ as effective gravitational equations on the brane ...
  - single bulk associated variable $E_{\mu\nu}$

    $\rightsquigarrow$ describes the permanent influence of bulk teory on brane-world gravity
  - *not a closed system*

    $\rightsquigarrow$ bulk solutions essential to fully describe the gravity induced on the brane

bulk equations system: $N = 1$

- two linear equations with two variables

  $g^{\mu\nu} T_{\mu\nu} = 0 \qquad \& \qquad W = 0$

- non-zero determinant

  $\rightsquigarrow$ $\{(h\pounds_n K) - (KK)\}$ and $\{{\pounds_n}^2\phi - a^e\nabla_e\phi\}$ can be determined uniquely

- $\{\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\}$ to be calculated subsequently

effective gravitational equations on the brane: $N = 1$, $d = 5$, $\mathbb{Z}_2$

- 4-dimensional brane

  embedded in a 5-dimensional space-time with $\mathbb{Z}_2$ symmetry

$\rightsquigarrow$ how shall we do it?

- take $T^{||||}_{\mu\nu} = 0$
- enter the solution of the brane limit of bulk equations system

  $\rightsquigarrow \{(h\pounds_n K) - (KK)\}$ & $\{{\pounds_n}^2\phi - a^e\nabla_e\phi\}$ & $\{\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\}$

  - slightly readjust the relative $R_{\mu\nu}$ vs. R coefficient with $T^{\perp\perp} = 0$

    $\rightsquigarrow$ 'Einstein-like' form with the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R\, h_{\mu\nu}$

- enter the result of junction conditions analysis

  $\rightsquigarrow [K_{\mu\nu}]_+$ & $[\pounds_n\phi]_+$

effective gravitational equations on the brane: $N = 1$, $d = 5$, $\mathbb{Z}_2$

$\rightsquigarrow$ effective gravitational equations on the brane read

$$G_{\mu\nu} + E_{\mu\nu} + \tfrac{2}{3}\Big((DD)_{\mu\nu}\phi - h_{\mu\nu}(DD)\phi\Big) + \tfrac{1}{4}h_{\mu\nu}(D\phi)^2 + h_{\mu\nu}\frac{V(\phi)}{2\alpha_1} +$$
$$+ \frac{1}{(2\alpha_1)^2}\Big[\tfrac{1}{3}\tau\tau_{\mu\nu} - (\tau\tau)_{\mu\nu} + h_{\mu\nu}\Big(\tfrac{1}{2}(\tau\tau) - \tfrac{1}{12}\tau^2 - \tfrac{1}{2}\tau\tau_\phi + \tfrac{3}{4}\tau_\phi^2\Big)\Big] = 0$$

Outline

Friedmann equations in first order dilaton gravity

- the effective equations on the brane derived . . . so what?

  $\rightsquigarrow$ let's see what can happen to the physics

- phenomenological applications to cosmology

  $\longrightarrow$ *Friedmann equations modified / generalized*

- standard cosmology $\rightsquigarrow$ FRW metric tensor ansatz $\longrightarrow$ Friedmann equations are given by gravitational equations of motion
  - trace
  - (t,t) component

  $\rightsquigarrow$ let's just do the very same here

modified Friedmann equations: comments and complaints?

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{\dot{a}}{a}\dot{\phi} - \frac{1}{3}\ddot{\phi} + \frac{1}{6}\dot{\phi}^2 + \frac{1}{3\alpha_1}V(\phi) + \\ + \frac{1}{(4\alpha_1)^2}\left[-\frac{2}{3}(\tau_1^1)^2 - 2(\tau_4^4)^2 + \frac{4}{3}\tau_1^1\tau_\phi + 4\tau_4^4\tau_\phi - 2\tau_\phi^2 \right] = 0$$

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} - \frac{1}{3}E_1^1 - \frac{2}{3}\frac{\dot{a}}{a}\dot{\phi} + \frac{1}{12}\dot{\phi}^2 + \frac{1}{6\alpha_1}V(\phi) + \\ + \frac{1}{(4\alpha_1)^2}\left[\frac{1}{3}(\tau_1^1)^2 - \frac{2}{3}\tau_1^1\tau_4^4 - (\tau_4^4)^2 + \frac{2}{3}\tau_1^1\tau_\phi + 2\tau_4^4\tau_\phi - \tau_\phi^2 \right] = 0$$

- part with the scalar factor only: exactly the same
- however, there are obviously quite relevant differences
  - terms associated with dilaton appear, as well as mixing terms

    $\rightsquigarrow$ due to the introduction of the additional field interacting with graviton
  - terms quadratic in the brane localized terms

    $\rightsquigarrow$ a feature of higher-dimensional theories?
  - $E_{\mu\nu}$: influence of the original higher-dimensional theory

    $\rightsquigarrow$ on the effective 4-dimensional phenomenology

Outline

effective gravitational equations on the brane: arbitrary N

- derivation procedure for the effective gravitational equations on the brane

  $\longrightarrow$ established

  $\leadsto$ works fine, we've just seen the example of $N = 1$ & $d = 5$

- and what about higher orders?

  $\longrightarrow$ exactly the same procedure, namely

  - take $T^{|||}_{\mu\nu} = 0$
  - enter the solution of the brane limit of bulk equations system

    $\leadsto \{(h\pounds_n K) - (KK)\}$ & $\{\pounds_n{}^2\phi - a^e\nabla_e\phi\}$ & $\{\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\}$
  - enter the result of junction conditions analysis

    $\leadsto [K_{\mu\nu}]_+$ & $[\pounds_n\phi]_+$

effective gravitational equations on the brane: arbitrary N

- derivation procedure for the effective gravitational equations on the brane
  $\longrightarrow$ established
  $\rightsquigarrow$ works fine, we've just seen the example of $N = 1$ & $d = 5$
- and what about higher orders?
  $\longrightarrow$ exactly the same procedure, namely
  - take $T^{||||}_{\mu\nu} = 0$
  - enter the solution of the brane limit of bulk equations system
    $\rightsquigarrow$ $\{(h\pounds_n K) - (KK)\}$ & $\{{\pounds_n}^2\phi - a^e\nabla_e\phi\}$ & $\{\pounds_n K_{\mu\nu} - (KK)_{\mu\nu}\}$
  - enter the result of junction conditions analysis
    $\rightsquigarrow$ $[K_{\mu\nu}]_+$ & $[\pounds_n\phi]_+$

conclusions

- starting point: higher order dilaton gravity

  $\longrightarrow$ natural to consider in higher-dimensional space-times

  $\rightsquigarrow$ physically viable equations of motion: *constructed*

  $\rightsquigarrow$ appropriate lagrangian: *presented*

- effective gravitational equations on the brane (co-dimension 1)

  $\longrightarrow$ derivation procedure for arbitrary order N: *established*

  $\rightsquigarrow$ *details* and *results* presented explicitly for $N = 1$ & $d = 5$

  $\hookrightarrow$ together with a *cosmological example*

  $\rightsquigarrow$ modified Friedmann equations on 4d brane