

# Generalised fluxes and de Sitter vacua

Beatriz de Carlos  
University of Southampton  
StringPheno, 18th June 2009

w/ A. Guarino, J.M. Moreno, almost done!

# Contents

1. Brief review of moduli stabilisation
2. Algebras and non-geometric fluxes
3. Looking for vacua using no-go theorems
4. Conclusions

# Moduli stabilisation

- Moduli are present in any string model
- Many of them parametrise physical quantities, i.e. must acquire a VEV if we are to obtain the Standard Model at low energies
- Their stabilisation is likely to be linked to the breakdown of SUSY
- They have potential cosmological interest

# Past history

Nilles'84

- Partial success in stabilising moduli through non-perturbative effects: **multiple gaugino condensation** in the heterotic

Krasnikov'87

Casas, Lalak, Munoz, Ross'90

- Minima that broke SUSY were **AdS**

BdC, Casas, Munoz '92

- Very steep potentials: **runaway dilaton**

Brustein, Steinhardt'93

# N=1, D=4 SUGRA

Scalar potential:

$$V_F = e^K \{ K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \}$$

K, Kähler potential, W superpotential

$$D_I W = K_I W + W_I \quad \text{Kähler derivative}$$

In general this is a **multivariable** potential

SUSY preserved:

$$D_I W = 0 \text{ for all } I$$

$$\text{AdS} \quad \rightarrow \quad V = -3e^K |W|^2$$

$$\text{Minkowski} \quad \rightarrow \quad V = W = 0$$

SUSY broken:

$$D_I W \neq 0 \text{ for some } I$$

AdS  
Minkowski  
dS

} all possible

but mostly **AdS** solutions found!

# Recent progress

Flux compactification in type IIB opens up a new path in model building

Dasgupta, Rajesh, Sethi'99

Gukov, Vafa, Witten'00

Giddings, Kachru, Polchinski'02

KKLT proposal: combine fluxes and np effects

Kachru, Kallosh, Linde, Trivedi'03

$$K = -3 \ln(T + \bar{T}), \quad W = W_0 + Ae^{-aT}$$

Gives a SUSY-preserving, AdS minimum

More realistic models in the heterotic and LVC give SUSY breaking with, again, AdS

Balasubramanian, Berglund, Conlon, Quevedo'05

BdC, Gurrieri, Lukas, Micu'06

# If we now forget about NP effects...

In type IIA it is possible to generate a superpotential for all (closed) string moduli just from fluxes

Grimm, Louis'05

Derendinger, Kounnas, Petropoulos, Zwirner'05

Villadoro, Zwirner'05

DeWolfe, Giryavets, Kachru, Taylor'05

Cámara, Font, Ibáñez'05

These can be **NS-NS** ( $H_3$ ), **R-R** ( $F_p$ ) and also **geometric** ( $\omega$ ) fluxes

Graña, Minasian, Petrini, Tomasiello'06

Andriot'08

Caviezel, Koerber, Kors, Lüst, Tsimpis, Zagermann '08

Aldazábal, Font'08

To recover T-duality between IIA and IIB we have to introduce **non-geometric** (Q,R) fluxes in the latter

Shelton, Taylor, Wecht '05, '06

For a symmetric orbifold,  $T^6/Z_2 \times Z_2$

$$K = -\ln(-i(S-Sb)) - 3\ln(-i(\tau-\tau b)) - 3\ln(-i(U-Ub))$$

$$W = P_1(\tau) + S P_2(\tau) + U P_3(\tau)$$

- **S** is the dilaton
- **U** is the Kähler (IIB), complex structure (IIA) modulus
- **$\tau$**  is the complex structure (IIB), Kähler (IIA) modulus

W is **linear** in S and U, whereas

$$P_1(\tau) = a_0 - 3a_1\tau + 3a_2\tau^2 - a_3\tau^3$$

$$P_2(\tau) = -b_0 + 3b_1\tau - 3b_2\tau^2 + b_3\tau^3$$

$$P_3(\tau) = 3(c_0 + (\hat{c}_1 + \check{c}_1 - \zeta_1)\tau - (\hat{c}_2 + \check{c}_2 - \zeta_2)\tau^2 - c_3\tau^3)$$

In **type IIB** language

$a_0, \dots, a_3$  given by R-R fluxes

$b_0, \dots, b_3$  given by NS-NS fluxes

$c_0, \dots, c_3$  given by Q fluxes ← **non geometric**

**They must be all integers**

# Comments on method

- The scalar potential is a function of 3 complex fields ( $S, U, \tau$ ), or 6 real variables
- It contains polynomials of high degree
- The stationary conditions,  $\partial V=0$ , are difficult to solve in general
- Most results in the literature look for SUSY solutions, solving the F-equations
- Powerful techniques based on computational algebra are available

Shelton, Taylor, Wecht'06

Micu, Palti, Tasinato'07

Font, Guarino, Moreno'08

Gray, He, Lukas, Ilderton'09

# Summarising so far

enormous efforts have been devoted to understanding flux compactification in string theory

non geometric fluxes provide a T-dual description between IIA and IIB

the resulting potential is a polynomial in the different fields and difficult to minimise

only SUSY and/or AdS solutions have been found

Goal: can we find SUSY breaking, dS solutions?

To achieve this we combine  
two different pieces of  
research

The classification of all subalgebras satisfied by  
Q fluxes in IIB (on  $T^6/Z_2 \times Z_2$ )

Font, Guarino, Moreno'08

A no-go theorem on the existence of de Sitter  
vacua and inflation in IIA

Hertzberg, Kachru, Taylor, Tegmark'07  
Zagermann's talk

# Generalised fluxes and algebras

Font, Guarino, Moreno'08

Consider IIB compactified on  $(T^2 \times T^2 \times T^2) / (Z_2 \times Z_2)$

NS-NS ( $H_3$ ) and Q fluxes can be regarded as **structure constants** of an extended (12d) symmetry algebra of the compactification

Shelton, Taylor, Wecht'06

Dabholkar, Hull'06

The algebra has isometry generators ( $Z_a$ ) and **gauge symmetry generators ( $X^a$ )**

$$[X^a, X^b] = Q_d^{ab} X^d, \quad [Z_a, X^b] = Q_a^{bd} Z_d, \quad [Z_a, Z_b] = H_{abd} X^d$$

Jacobi identities ( $H_3 Q = 0$ ,  $Q^2 = 0$ ) and tadpole cancellation conditions restrict the possible values of the flux constants ( $a_s, b_s, c_s$ )

Even more,  $Q$  can only be one of 5 possible 6d subalgebras:

- $SO(4) \sim SU(2)^2$
- $SO(3,1)$
- $SU(2) + U(1)^3$
- $iso(3) \sim SU(2) \oplus U(1)^3$
- $nil \sim U(1)^3 \oplus U(1)^3$

# Parameter counting

$K_1, K_2$  define the 6d subalgebra

$\varepsilon_1, \varepsilon_2$  define the embedding in the 12d algebra

$\zeta_3, \zeta_7$  define the localised sources

Moreover we can perform redefinitions of fields/couplings to end up with

$$W = W(\varepsilon_2/\varepsilon_1, \zeta_7/\zeta_3)$$

the **F-equations** can be solved analytically

# No-go theorem and inflation

Hertzberg, Kachru, Taylor, Tegmark'07

Instead of looking at  $V$  in terms of  $W$  and  $K$ ,  
let's write it in terms of the contributions from  
the different fluxes in IIA

$$V = V_{H_3} + V_{F_p} + V_{D6} + V_{O6}$$

$$V_{H_3} \sim 1/y^3 \sigma^2$$

$$V_{F_p} \sim 1/y^{3-p} \sigma^4$$

$$V_{D6} \sim 1/\sigma^3$$

$$V_{O6} \sim -1/\sigma^3$$

$$\sigma = \text{Im}(S), \quad y = \text{Im}(\tau)$$

$V_{H_3}$  and  $V_{F_p}$  are positive definite

This potential satisfies

$$-\gamma \partial V / \partial \gamma - 3\sigma \partial V / \partial \sigma = 9V + \sum p V_{F_p}$$

Then, at an extremum,  $V < 0$ !

Way out: consider geometric ( $V_\omega$ ) and non geometric ( $V_Q, V_R$ ) fluxes. The previous condition reads

$$\begin{aligned} & -\gamma \partial V / \partial \gamma - 3\sigma \partial V / \partial \sigma = \\ & -2 V_\omega - 4V_Q - 6V_R + 9V + \sum p V_{F_p} \end{aligned}$$

$V_\omega$  used to construct de Sitter vacua

# Our work

BdC, Guarino, Moreno'09

We consider N=1 orientifolds of the  $Z_2 \times Z_2$  orbifold

- it is its own mirror under T-duality (U and  $\tau$  swap roles)
- IIA and IIB compactified on these structures are equivalent under T-duality

IIB with O3/O7  $\leftrightarrow$  IIA with O6  $\leftrightarrow$  IIB with O5/O9

# Strategy:

i) we have a complete classification of allowed fluxes in **IIB** (based on the  $Q$  subalgebra)

$$W = W(\epsilon_2/\epsilon_1, \zeta_7/\zeta_3)$$

ii) we can map this **IIB** potential to a **IIA** one and use no-go theorems to look for de Sitter vacua

Moreover: it seems that only compactifications on  $Z_2 \times Z_2$  may allow for inflation

# The generalised scalar potential

We work with real fields,

$$S = s + i \sigma, U = t + i \mu, \tau = x + i y$$

in terms of which the potential reads

$$V = A(y, \mu, \phi) / \sigma^2 + B(\mu) / \sigma^3 + C(y, \mu, \phi) / \sigma^4$$

$\phi=(s,t,x)$  are the axions and  $\mu$  (IIB)  $\rightarrow \mu/\sigma$  (IIA)

A accounts for generalised fluxes ( $H_3, R, Q, \omega$ )

B accounts for localised sources

C accounts for R-R ( $F_p$ ) fluxes

We can now study moduli stabilisation in a **systematic way**, relating  $A, B, C$  to the original flux parameters in IIB

This already tells us the signs of the different contributions, **facilitating** the search for de Sitter vacua

Most of the search can be done **analytically** because  $S$  and  $U$  enter  $W$  linearly.

$$\partial V / \partial s = 0 \Rightarrow s_0 = s_0(x_0, y_0)$$

$$\partial V / \partial t = 0 \Rightarrow t_0 = t_0(x_0, y_0)$$

The physical parts of  $S, U$  ( $\sigma, \mu$ ) can be stabilised analytically by imposing  $V=0$

$$\sigma_0 = \sigma_0(x_0, y_0)$$

$$\mu_0 = \mu_0(x_0, y_0)$$

We are left with  $\partial V / \partial x = \partial V / \partial y = 0$

After replacing all other fields these are two nasty equations that require **numerical analysis**

# Results

Of the 5 possible subalgebras, **four** of them do not give de Sitter vacua

At most one can have Minkowski/de Sitter minima with one **tachyonic** direction

**$SO(3,1)$**  contains plenty of de Sitter vacua with all moduli stabilised

# Results

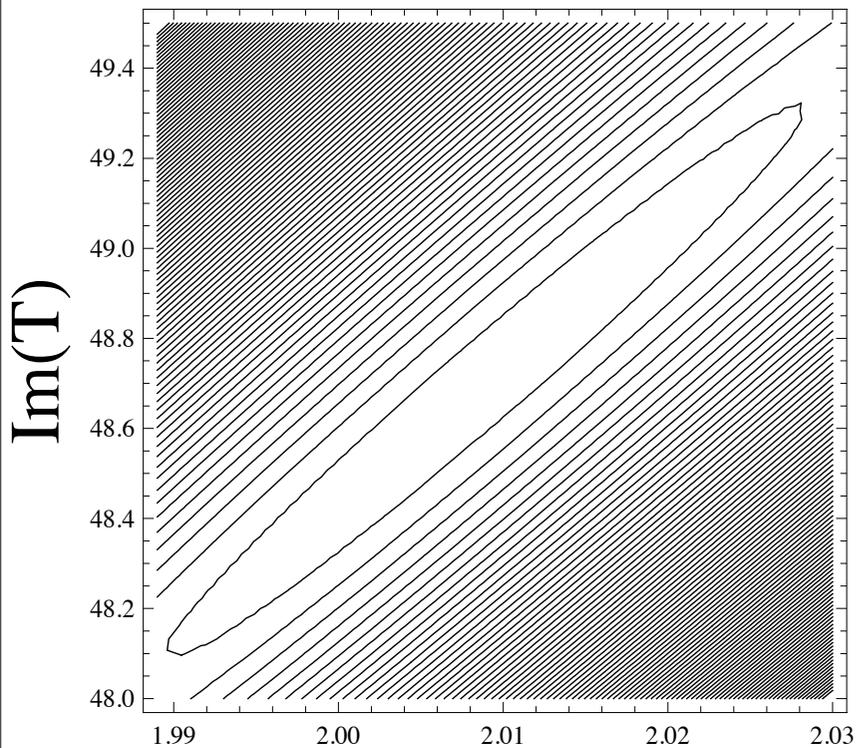
subalgebra	SO(3,1)		SO(4)		iso(3)		nil		SU(2)+U(1) <sup>3</sup>	
class	NG		NG		G		G		G	NG
no-go	✓	✗	✓	✗	✓	✗	✓	✗	✗	✓
dS vacua	✓	✗	✗		✗		✗		✗	

NG/G: admits a description in terms of non geometric/geometric backgrounds

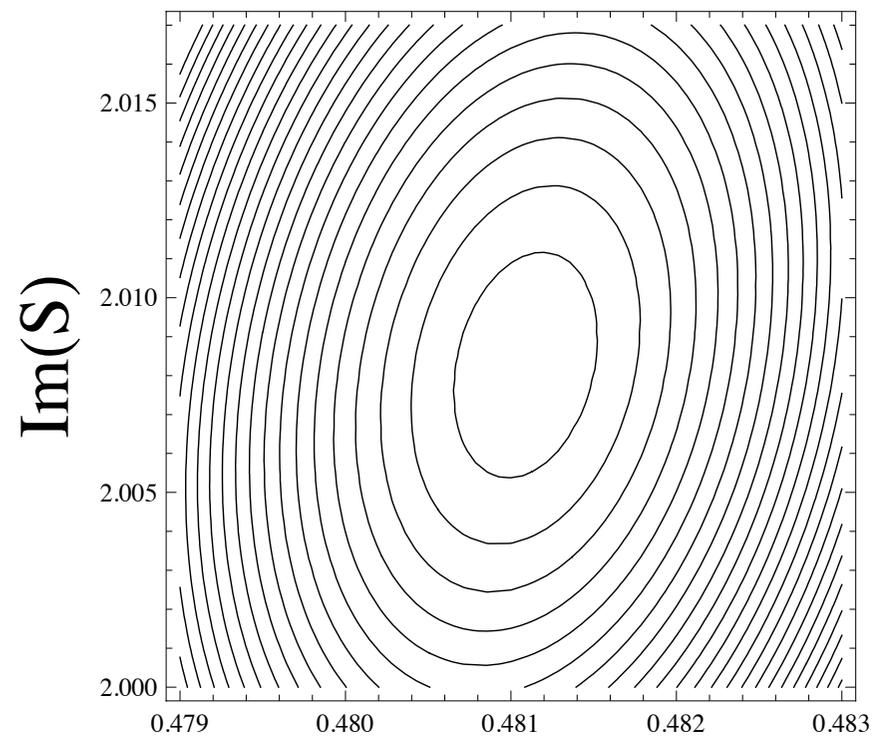
Villadoro, Zwirner'05

Font, Guarino, Moreno'08

Dall'Agata, Villadoro, Zwirner'09

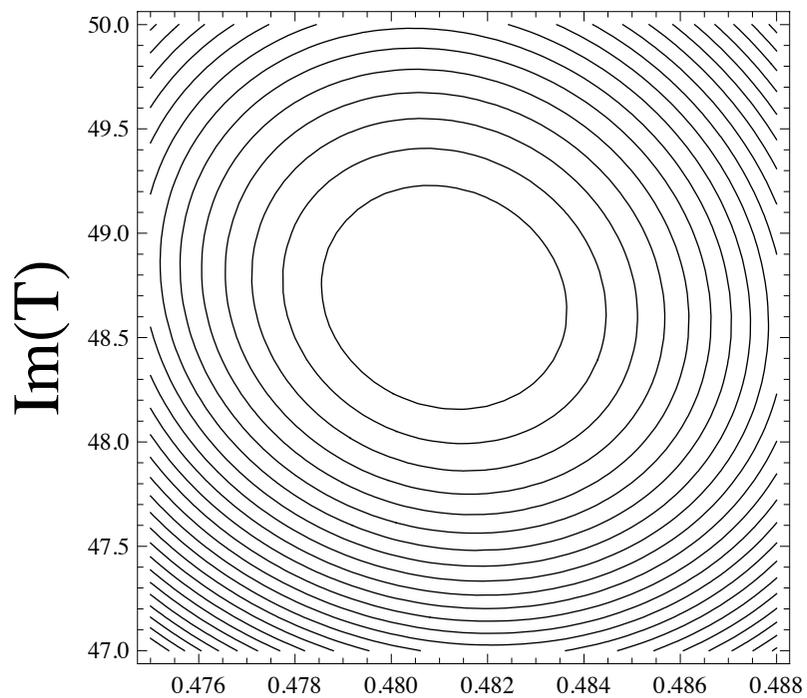


**SO(3,1)**



**Im(U)**

**Im(S)**

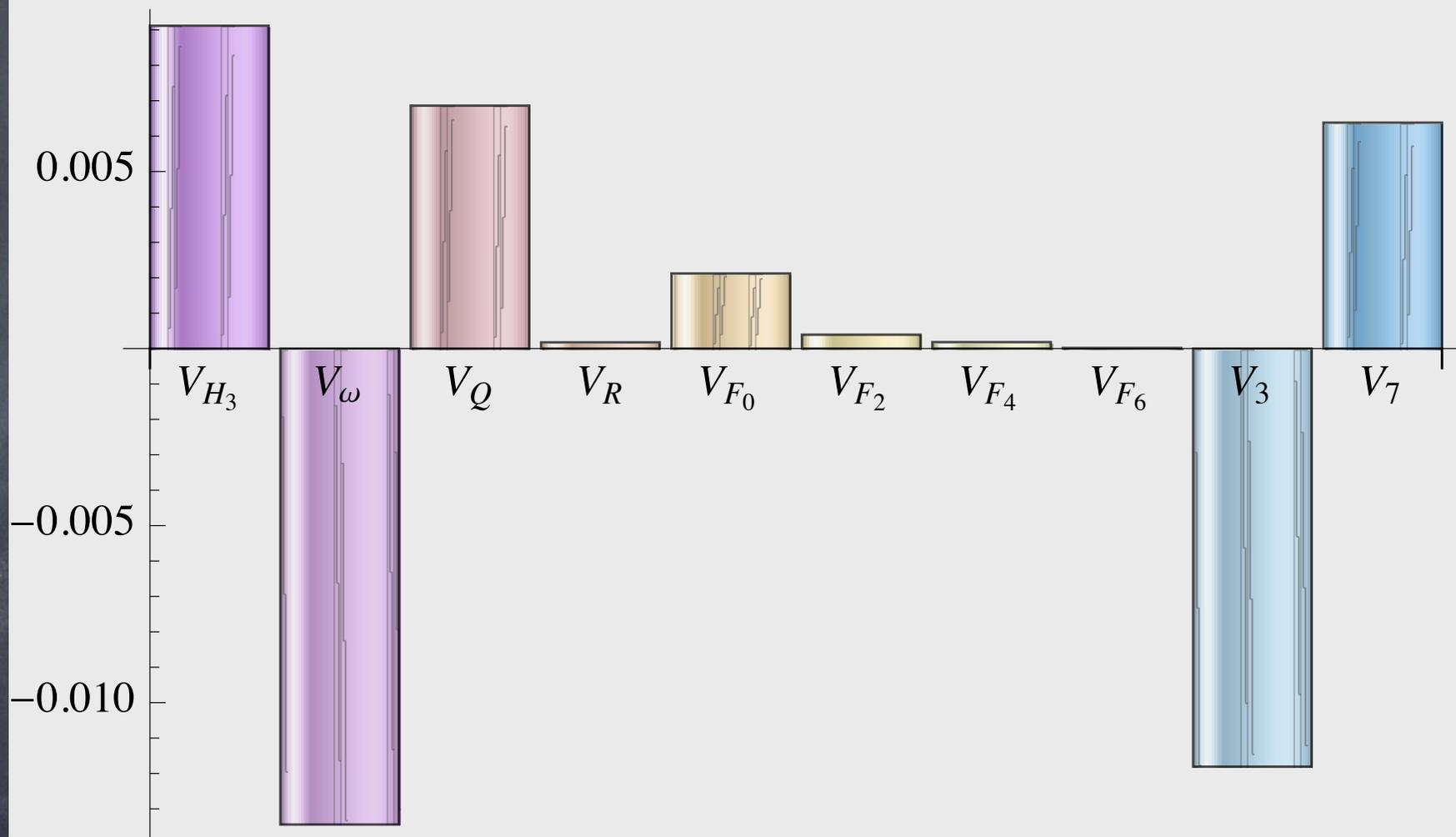


**Im(U)**

$$\zeta_7=16$$

$$\varepsilon_2=44.309$$

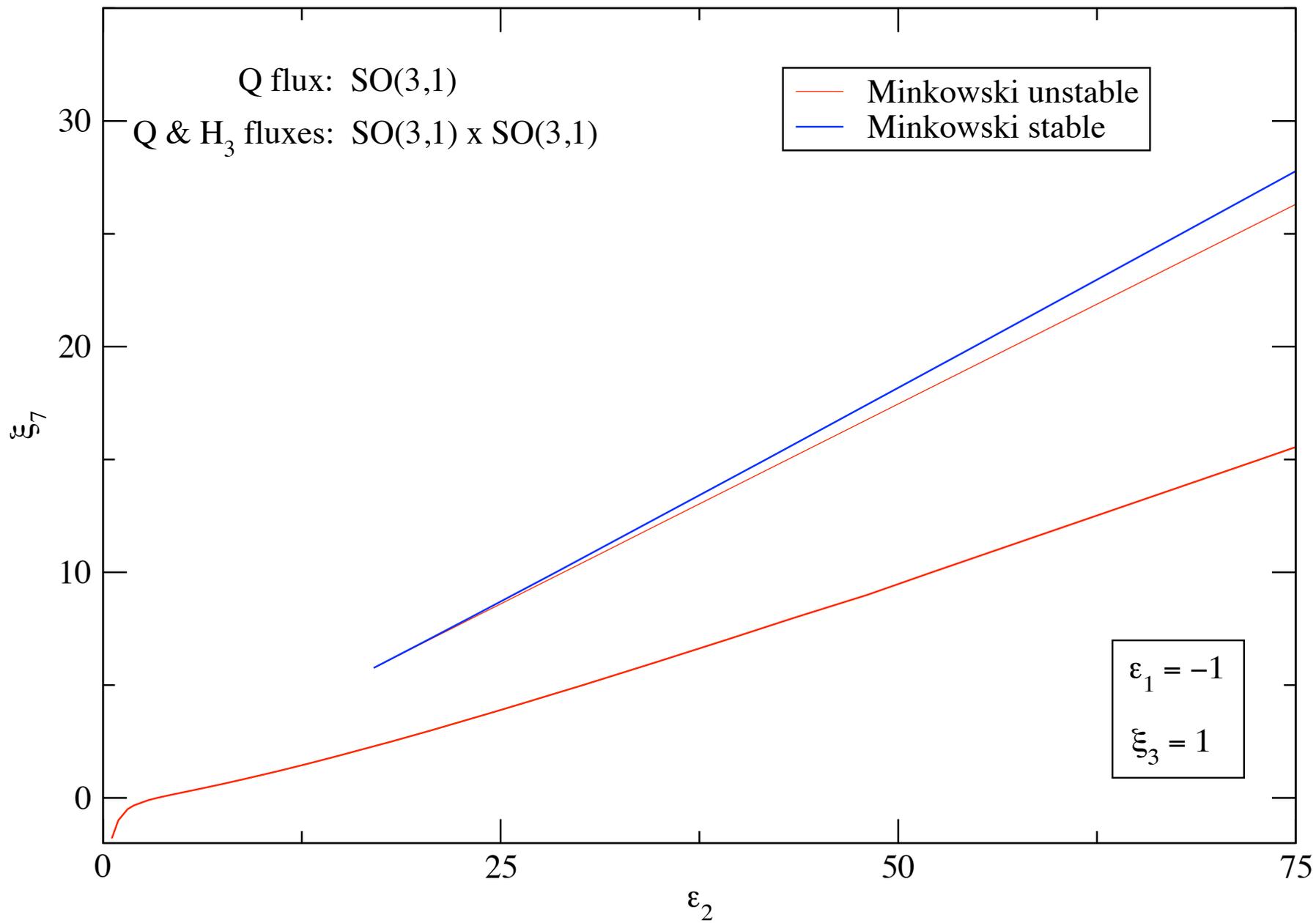
# Scalar potential contributions

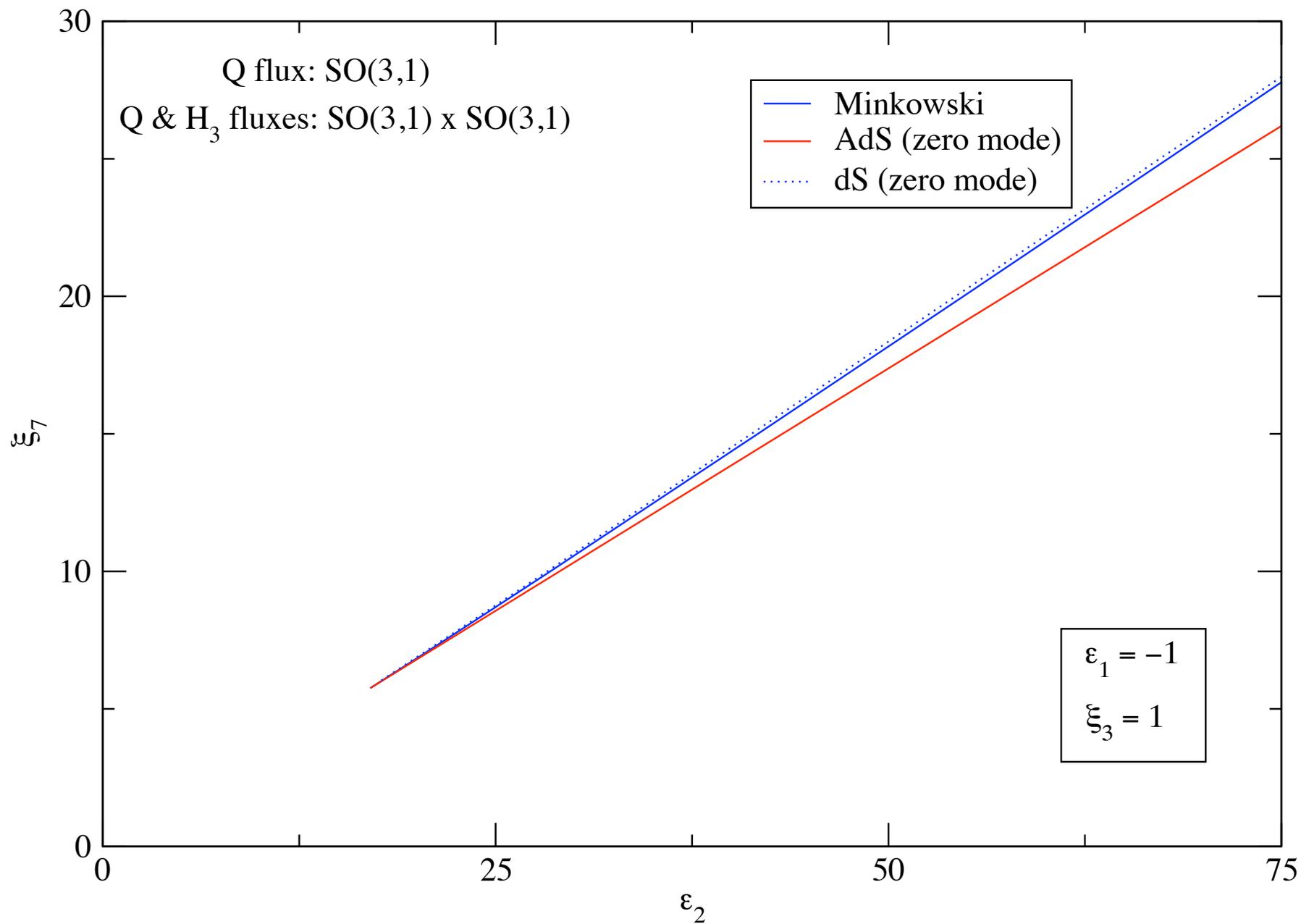


$SO(3,1)$

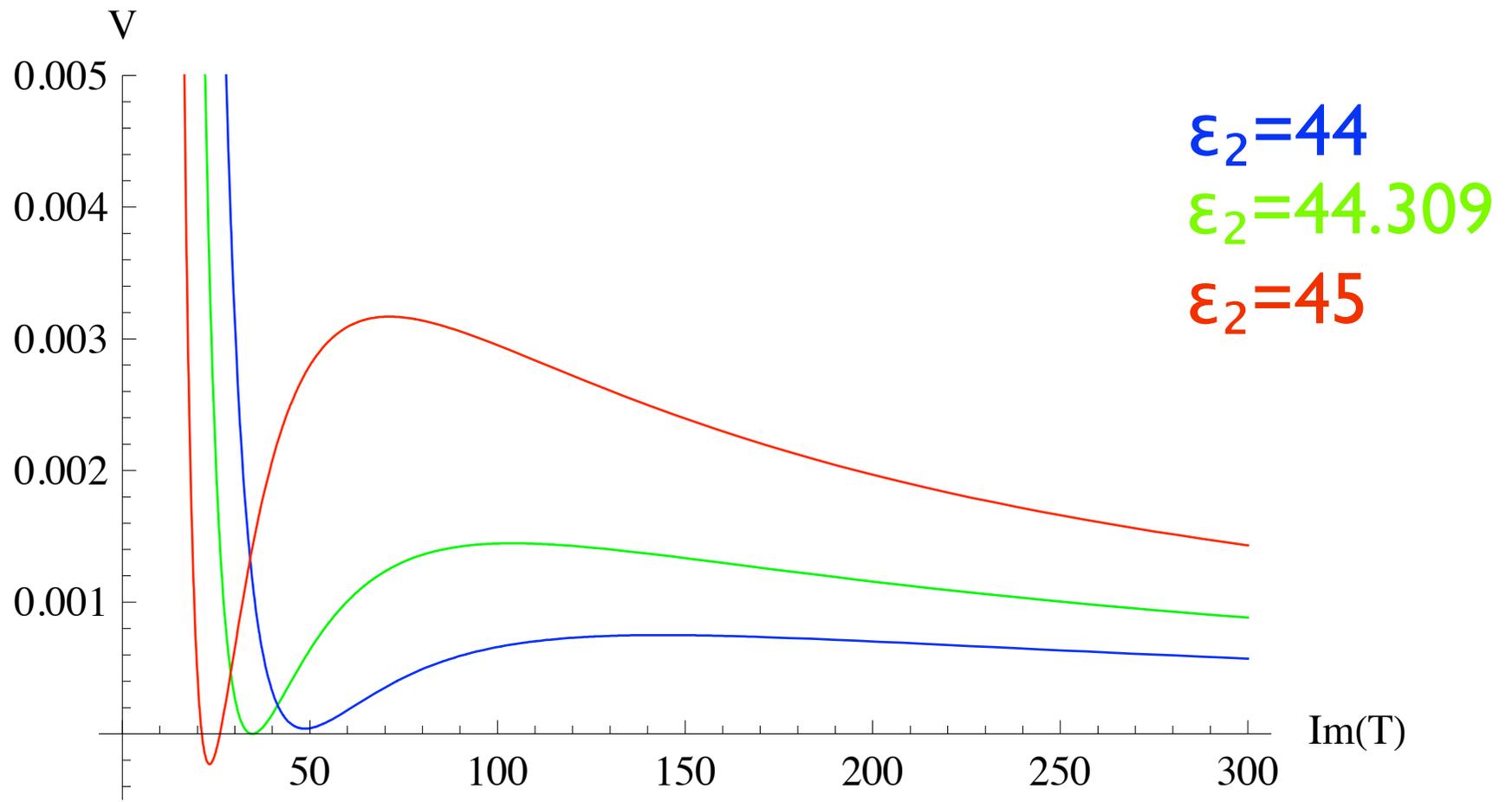
$\zeta_7=16$

$\varepsilon_2=44.309$





The vacua oscillate between AdS and dS according to the value of  $\epsilon_2$  (for fixed  $\zeta_7$ )



# Conclusions

- We have discussed T-dualities between IIA and IIB using **non geometric fluxes**
- The resulting  $W$  can stabilise all moduli, but the potential is quite **involved**. Treating it requires new, systematic, **analytic** and **numerical** methods
- Strategy: use **algebraic** results in IIB, which simplifies  $W$  and the number of fluxes, and **no-go theorems** on the existence of de Sitter vacua in IIA
- This results in a **systematic** and **feasible** search which gives plenty of de Sitter (SUSY breaking) minima
- The method is exportable to other orbifolds