Dynamical SUSY breaking from unoriented D-brane instantons

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Plan

- Dynamical SUSY breaking: a lightning review
- Unoriented $\mathbf{Z}_n$ quivers and D-brane instantons
- Non-perturbative dynamics of $U(5) \times U(1)$ $\mathbf{Z}_5$-quiver theory
  - Perturbative supersymmetric vacua, FI terms, instantons at weak coupling and SUSY breaking
  - Macroscopic description: mesons and baryons, deformation of moduli space, dynamical SUSY breaking
  - (Anomaly) matching between strong and weak couplings
  - D-instanton calculus of the superpotential
- Summary and outlook

Based on 0904.2156 with F. Fucito and J. F. Morales,
see also M. B., S. Kovacs and G. C. Rossi hep-th/0703142 for review

Dynamical Supersymmetry Breaking

Non-perturbative quantum effects, hierarchically small scales

$$\Lambda = M e^{-8\pi^2/\beta g^2(M)} << M$$

Chiral condensates (*e.g.* gaugino $\langle\lambda\lambda\rangle$) may be incompatible with supersymmetry and Konishi anomaly in chiral models without flat directions *e.g.* $SU(5)$ GUT with one or two generations of $\mathbf{10} + \mathbf{5}^*$

Effective description of non-supersymmetric vacuum and excitations at 'strong coupling' (problematic / questionable)

Use Seiberg duality to concoct weakly coupled models

Meta-stable states, ..., Retrofitting, ..., Gauge/gravity mediation

String embedding $\rightarrow$ unoriented D-brane instantons, field dependent (gauge) couplings, hidden sectors, ...

Two classes of D-brane instantons

[Ibanez, Uranga; Blumenhagen, Cvetic, Weigand; MB, Kiritsis; Billò, Frau, Lerda, Ferretti; MB, Fucito, Morales; Argurio, Bertolini, Kachru; Camara, Dudas, Maillard, Pradisi; Angelantonj, Schmidt-Sommerfeld, Akerblom, ...]

- 'Gauge' instantons: $F = *_4 F$, EDp-branes wrapping the same cycle C as a stack of background D(p+4)-branes, strength

$$e^{-W_{p+1}(C)/g_s \ell_s^{p+1}} = e^{-8\pi^2/g^2_{YM}}$$

roughly speaking 4 N-D directions (spacetime)

- 'Exotic' instantons: $F \wedge F = *_8(F \wedge F)$, EDp'-branes wrapping cycle C' not wrapped by any stack of background D(p+4)-branes, strength

$$e^{-W_{p'+1}(C')/g_s \ell_s^{p'+1}} \neq e^{-8\pi^2/g^2_{YM}}$$

roughly speaking 8 N-D directions (spacetime + internal)

Unoriented Quiver Theories

D-branes at $\mathbf{C}^3/\mathbf{Z}_n$ singularity: $(z_1, z_2, z_3) = (\omega^{k_1} z_1, \omega^{k_2} z_2, \omega^{k_3} z_3)$,
SUSY: $k_1 + k_2 + k_3 = 0 \ (mod\ n)$
Action on Chan-Paton factors $\rho(\mathbf{Z}_n) \leftrightarrow$ 'fractional' D-branes,
Quiver Gauge Theory ($\alpha' \approx 0$): Vectors V in the $N_i \bar{N}_i$ of $U(N_i)$,
Chirals Φ_i in the $N_j \bar{N}_l$ with $k_i + j - l = 0 \ (mod\ n)$
Tree level superpotential inherited from $\mathcal{N} = 4$ SYM

$$W = Tr(\Phi_1[\Phi_2, \Phi_3])$$

Twisted RR tadpole cancellation $\leftrightarrow$ NO chiral anomaly, $\beta_i = 0$
Unoriented projections: $\Omega^{\pm}$-planes $\rightarrow$ SO/SP groups and $\beta_i \neq 0$
Non-perturbative superpotential (schematically)

$$W_{n-p} = \Lambda^{K_i\beta_i}/\phi^{K_i\beta_i - 3} \quad \text{with} \quad \Lambda^{\beta_i} = M_s^{\beta_i} e^{-T(C_i)}$$

generated by 'gauge' instantons if $dim \mathfrak{M}^{fermi}_{K_i,N_i} = 2K_i\beta_i - 4$
'Exotic' D-brane instantons generate $W_{C,n} = M_s^{3-n} e^{-a_n T_C} \phi_C^n$

$U(5) \times U(1)$ GUT-like from the $\mathbf{Z}_5$-quiver

Chiral matter content

fields	$SU(2)_F$	$SU(5)$	$U(1)_1 \times U(1)_5$
A^i_{uv}	$\mathbf{2}$	$\mathbf{10}$	$(0,2)$
B^i	$\mathbf{2}$	$\mathbf{1}$	$(-1,0)$
C^{iu}	$\mathbf{2}$	$\mathbf{5}^*$	$(1,-1)$
C^{3u}	$\mathbf{1}$	$\mathbf{5}^*$	$(-1,-1)$
E_u	$\mathbf{1}$	$\mathbf{5}$	$(0,1)$

Tree level superpotential ($SU(2)_F$ invariant)

$$W_{tree} = C^{iu} B_i E_u + C^{iu} A_{iuv} C^{3v}$$

Gauge couplings: $\beta_{SU(5)} = 10$, $\Lambda_{SU(5)} = M_s e^{a_o S + a_1 T_1 + a_2 T_2}$

T_i twisted moduli: axions $\tau_i = Im T_i$ shift under $U(1)_1 \times U(1)_5$

FI terms: $\xi_5 = a_1 t_1 + a_2 t_2$ and $\xi_1 = b_1 t_1 + b_2 t_2$

with $t_i = Re T_i$ blow-up modes of $\mathbf{Z}_5$ singularity

Perturbative analysis

Classical F-terms

$$
\begin{aligned}
F_{B_i} &= C^{iu} E_u \\
F_{A_i} &= C^{iu} C^{3v} \\
F_{C^3} &= A_{iuv} C^{iu} \\
F_{C^i} &= B_i E_u + A_{iuv} C^{3v}
\end{aligned}
$$

$U(1) \times U(5)$ D-terms

$$
\begin{aligned}
D &= -|B_i|^2 + |C^{\alpha u}|^2 - |C^{3u}|^2 + \xi_1 \\
D^u_v &= \bar{A}^{uw}_i A^i_{wv} - \bar{C}_{iv} C^{iu} - \bar{C}_{3v} C^{3u} + \bar{E}^u E_v + \xi_5 \delta^u_v
\end{aligned}
$$

For $\xi_5 = 0$, $\xi_1 > 0$, $SU(5)$ preserving, perturbative supersymmetric (degenerate) vacuum

$$
A^i = C^I = E = 0 \qquad |B_1|^2 + |B_2|^2 = \xi_1
$$

Including instantons at weak coupling

Despite $dim\mathfrak{M}_k^{fermi} = k(10_\lambda + 3_{\psi_C} + 1_{\psi_E} + 6_{\psi_A}) = 20k$ and $\beta = 10$ do not satisfy condition for instanton superpotential

$$2k\beta - 4 \neq dim\mathfrak{M}_k^{fermi}$$

one-instanton dominated correlator

$$\langle X^I(C,E)\, Z^{iJ}(A,C)\, Z^{jK}(A,C)\rangle = \epsilon^{IJK}\,\epsilon^{ij}\,\Lambda^{10}$$

can be computed in weak coupling regime, where

$$X^I(C,E) = C^{Iu}E_u \quad , \quad Z^{il}(A,C) = \frac{1}{12}\epsilon^{u_1\dots u_5} A^i_{u_1u_2}\, A^j_{u_3u_4}\, A_{u_5v,j}\, C^{Iv}$$

Konishi anomaly for $\phi = E \to$ gaugino condensate

$$\langle\lambda\lambda\rangle \approx B_i\langle X^i(C,E)\rangle$$

For $\xi_1 = m^2$, take *e.g.* $B_1 = m$, mass for C^{1u} and E^u from W_{tree}.
For large $m >> \Lambda$, C^{1u} and E^u decouple $\rightarrow$

$U(5)$ GUT with 2 generations of $(\mathbf{10} + \mathbf{5}^*)$ and $\hat{\beta} = 11$

Since $dim \hat{\mathfrak{M}}^{fermi}_{k=1} = 18 = 2\hat{\beta} - 4$, non-perturbative superpotential generated by 'gauge' instantons, uniquely fixed by dimensional analysis and $SU(5) \times SU(2)_F$ invariance

$$W_{n-p} = \frac{\hat{\Lambda}^{11}}{Z^{i\alpha}(A, C) Z_{i\alpha}(A, C)}$$

where $\hat{\Lambda}^{11} = m\Lambda^{10}$ and $Z^{i\alpha} = \epsilon^{u_1 \ldots u_5} A^i_{u_1 u_2} \, A^j_{u_3 u_4} \, A_{u_5 j, k} \, C^{\alpha v}$, with $\alpha = 2, 3 \neq 1$ labelling two $C^{\alpha u}$ remaining massless

$$W_{eff} = A^1_{uv} \, C^{2u} \, C^{3v} + \frac{\hat{\Lambda}^{11}}{Z^{i\alpha}(A, C) Z_{i\alpha}(A, C)}$$

No solution to $F_{I_o} = 0$, SUSY broken by instatons!

Alternatively, Konishi anomaly equation

$$\frac{1}{4}\bar{D}^2(\Phi^r e^{gV}\Phi^\dagger_s) = \Phi^r \frac{\partial W_{tree}}{\partial \Phi^s} + \frac{g^2}{32\pi^2}\delta^r{}_s tr_{R_s}(W^\alpha W_\alpha)$$

Since $\bar{Q}$'s annihilate supersymmetric states, VEV of left-hand side should be zero in a supersymmetric vacuum.
Take diagonal term (no sum over s)

$$\langle \phi^s \frac{\partial W_{tree}}{\partial \phi^s}\rangle + \frac{g^2}{32\pi^2}\, Tr_{R_s}\langle\lambda\lambda\rangle = 0$$

first term cancels for $\phi_s = A^2$ (absent from tree level superpotential), then non-zero gaugino condensate $Tr_{R_s}\langle\lambda\lambda\rangle \neq 0$ $\rightarrow$ supersymmetry dynamically broken.

Strong coupling description

'Mesons' and 'Baryons' (gauge-invariant composites)

$$
\begin{aligned}
X^I &= C^{Iu} E_u \\
Y^i_I &= \tfrac{1}{2}\epsilon_{IJK}\, A^i_{u_1 u_2}\, C^{Ju_1}\, C^{Ku_2} \\
\tilde{Y}^{ij} &= \tfrac{1}{4}\epsilon^{u_1\dots u_5} A^i_{u_1 u_2}\, A^j_{u_3 u_4}\, E_{u_5} \\
Z^{iI} &= \tfrac{1}{12}\epsilon^{u_1\dots u_5} A^i_{u_1 u_2}\, A^j_{u_3 u_4}\, A_{u_5 v, j}\, C^{Iv}
\end{aligned}
$$

Algebraic relations and quantum deformation of moduli space

$$Y^i_I\, Z^I_i = 0 \quad , \quad \epsilon_{IJK}\, X^I\, Z^J_i\, Z^{iK} + Y_{iI}\, \tilde{Y}^{ij} Z^I_j = 0 \to \Lambda^{10} \quad (\Delta = \beta = 10)$$

Counting of d.o.f. $18 = 42_{fields} - 24_{gauge} = 20_{compo's} - 2_{relat's}$ OK!
Effective superpotential

$$W_{eff} = B_i\, X^i + \delta^I_i\, Y^i_I + V\, Y^i_I\, Z^I_i + U\,(Y^i_I\, \tilde{Y}_{ij} Z^{Ij} + \epsilon_{IJK} X^I\, Z^J_i\, Z^{iK} - \Lambda^{10})$$

Similar to SQCD with $N_c = N_f$, where $B\tilde{B} - det(M) = \Lambda^{2N_c}$

The supersymmetric vacuum

F-terms $F_s = \partial W_{eff} / \partial \Phi_s$ with $\Phi_s = (B, X, Y, \tilde{Y}, Z, U, V)$.

$$
\begin{aligned}
F_{B_i} &= X^i \\
F_{X^I} &= B_i \delta^i_I + U\, \epsilon_{IJK}\, Z^J_i\, Z^{iK} \\
F_{Y^i_I} &= \delta^I_i + V\, Z^I_i + U\, \tilde{Y}_{ij}\, Z^{jI} \\
F_{\tilde{Y}^{ij}} &= U\, Y_{I(i}\, Z^I_{j)} \\
F_{Z^I_i} &= V\, Y^i_I + U\, (Y_{Ij}\, \tilde{Y}^{ij} + 2\, \epsilon_{IJK} X^K\, Z^{iJ}) \\
F_U &= Y_{Ii}\, \tilde{Y}^{ij} Z^I_j + \epsilon_{IJK}\, X^I\, Z^J_i\, Z^{iK} - \Lambda^{10} \\
F_V &= Y^i_I\, Z^I_i
\end{aligned}
$$

One-parameter (Z^{12} free) solution to $F_s = 0$

$$Z^{21} = -Z^{12} \qquad X^3 = \Lambda^{10}/[2(Z^{12})^2] \qquad V = 1/Z^{12}$$

all other composites and B_i set to zero, $W_{eff} = 0$ along the valley.

Supersymmetry breaking

For $B_i = (m, 0)$, eight (=32-24) 'massless' degrees of freedom: singlets B_i, gauge-invariants $Z^{i\alpha}$ and $Y_1^i = A^i_{u_1 u_2}\, C^{\alpha u_1}\, C^{u_2}_\alpha$.
Solve F-flatness conditions for 'massive' composites

$$X^1 = \frac{\Lambda^{10}}{Z^{i\alpha} Z_{i\alpha}} \qquad X^2 = \frac{Y_1^2}{m} \qquad U = -\frac{m}{Z^{i\alpha} Z_{i\alpha}} \qquad V = \frac{Z^{23}}{Z^{i\alpha} Z_{i\alpha}}$$

$$\tilde{Y}_{22} = -\frac{2Z^{13}}{m} \qquad \tilde{Y}_{12} = \frac{Z^{23}}{m}$$

all remaining ones set to zero, and plug into $W_{eff} \rightarrow$

$$W_{eff_o} = Y_1^1 + \frac{m\Lambda^{10}}{Z^{i\alpha} Z_{i\alpha}}$$

in perfect agreement with previous analysis at weak coupling!
F-flatness conditions for 'massless' fields cannot all be simultaneously satisfied $\rightarrow$ *supersymmetry is dynamically broken*

Anomaly matching conditions

Anomalies of global symmetries in 'microscopic' theory (weakly coupled elementary fields in UV) (should and do) match those of baryons and mesons in 'macroscopic' theory (IR, strongly coupling)
Original global symmetry (partially broken by W_{tree})

$$SU(3)_C \times SU(2)_A \times U(1)_A \times U(1)_C \times U(1)_E$$

Solution preserving largest possible symmetry

$$X^3 = \Lambda^2 \qquad Z^{iI} = \frac{1}{\sqrt{2}}\Lambda^4 \epsilon^{iI} \qquad I = 1, 2$$

with all remaining composite fields set to zero.
Tables summarize charges of elementary and composite fields under residual $G_F = SU(2) \times U(1)_M \times U(1)_N$ symmetry

$SU(2) \times U(1)^2$ charges of elementary and composite fields

Elementary fields

fields	A^i_{uv}	C^{iu}	C^{3u}	E_u
$SU(2)$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_M$	1	-3	0	0
$U(1)_N$	0	0	1	-1
$SU(5)$	$\mathbf{10}$	$\mathbf{5}^*$	$\mathbf{5}^*$	$\mathbf{5}$

Composite fields

fields	X^3	X^i	Y^i_3	Y^i_j	$\tilde{Y}^{(ij)}$	Z^{i3}	Z^{ij}	U	V
$SU(2)$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}, \mathbf{1}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{3}, \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$U(1)_M$	0	-3	-5	-2	2	3	0	0	2
$U(1)_N$	0	-1	0	1	-1	1	0	0	-1

$U(1)_{M,N}$ are (approximate) quantum symmetries $\leftarrow$
no $U(1)_{M,N}$ anomaly:

$$I_{U(1)_a SU(5)^2} = \sum_s dim(\mathbf{R}^s_{SU(2)})\, Q_a(\Phi_s)\ell(\mathbf{R}^s_{SU(5)}) = 0,$$

where $a = M, N$ and s runs over all elementary fields, *viz.*

$$\begin{aligned} I_{U(1)_M SU(5)^2} &= 2\tfrac{3}{2} + 2\,\tfrac{1}{2}(-3) = 0 \\ I_{U(1)_N SU(5)^2} &= \tfrac{1}{2}(1-1) = 0 \end{aligned}$$

Yet, **Cubic** anomalies of the global currents

$$\begin{aligned} I_{U(1)^n_M U(1)^{3-n}_N} &= \sum_s dim(\mathbf{R}^s_{SU(2)\times SU(5)})\ Q_M(\Phi_s)^n\, Q_N(\Phi_s)^{3-n} \\ I_{U(1)_a SU(2)^2} &= \sum_s dim(\mathbf{R}^s_{SU(5)})\, Q_a(\Phi_s)\ell(\mathbf{R}^s_{SU(2)}) \qquad a = M, N \end{aligned}$$

No cubic anomaly with $SU(2)$ currents only

No cubic anomalies with $U(1)_N$ in 'microscopic' description (non chiral), agreement with 'macroscopic' baryons/mesons (X, Y, Z)

$$
\begin{aligned}
I^{XYZ}_{U(1)^3_N} &= (-1)^3(2+3+1)+(1)^3(4+2)=0 \\
I^{XYZ}_{U(1)^2_N U(1)_M} &= 2(-3)+4(-2)+3(2)+2(3)+(2)=0 \\
I^{XYZ}_{U(1)_N U(1)^2_M} &= 2(-9)+4(4)+3(-4)+2(9)+(-4)=0 \\
I^{XYZ}_{U(1)_N SU(2)^2} &= \tfrac{1}{2}(-1+1)+2(1-1)=0
\end{aligned}
$$

Non-trivial anomalies in the microscopic theory

$$
\begin{aligned}
I^{ACE}_{U(1)^3_M} &= 20\,(1)^3-10(-3)^3=-250 \\
I^{ACE}_{U(1)_M SU(2)^2} &= 10\tfrac{1}{2}-15\tfrac{1}{2}=-\tfrac{5}{2}
\end{aligned}
$$

perfectly match those in the macroscopic theory

$$I^{XYZ}_{U(1)^3_M}=2(-3)^3+2(-5)^3+4(-2)^3+3(2)^3+4(3)^3+2=-250$$

$$I^{XYZ}_{U(1)_M SU(2)^2}=\tfrac{1}{2}(-3-5+3)+(2-2)=-\tfrac{5}{2}$$

D-instanton description

sector	fields	$SU(2)_F$	$SU(5)$	$U(1)_1 \times U(1)_5 \times U(1)_{inst}$
D3D3	A^i_{uv}	**2**	**10**	$(0,2,0)$
	B^i	**2**	**1**	$(-1,0,0)$
	C^{iu}	**2**	$\mathbf{5}^*$	$(1,-1,0)$
	C^{3u}	**1**	$\mathbf{5}^*$	$(-1,-1,0)$
	E_u	**1**	**5**	$(0,1,0)$
D(-1)D(-1)	$(a_{\alpha\dot\alpha}, \Theta^0_\alpha, \bar\Theta^{\dot\alpha}_0)$	**1**	**1**	$(0,0,0)$
	$(\bar\chi_i, \bar\Theta^{\dot\alpha}_i)$	**2**	**1**	$(0,0,-2)$
	χ^i	**2**	**1**	$(0,0,2)$
D(-1)D3	$(\bar w^u_{\dot\alpha}, \bar\nu^{0u})$	**1**	**5**	$(0,1,-1)$
	$(w^{\dot\alpha}_u, \nu^0_u)$	**1**	$\mathbf{5}^*$	$(0,-1,1)$
	ν^{iu}	**2**	**5**	$(0,1,1)$
	$\bar\nu^i$	**2**	**1**	$(1,0,-1)$
	$\bar\nu^3$	**1**	**1**	$(-1,0,-1)$
	ν^3	**1**	**1**	$(0,0,1)$

D-instanton superpotential

Supermeasure charges, $d\mathfrak{M} \to (-1, -10)$ compensate Λ^{10}

$$\int d^4x d^2\Theta\, W(\Phi_s) = \Lambda^{10} \int d\mathfrak{M} e^{-\mathcal{S}_B - \mathcal{S}_F}$$

with

$$\begin{aligned}
\mathcal{S}_F &= \bar{\Theta}_{0\dot{\alpha}}(w_u^{\dot{\alpha}} \bar{\nu}^{0u} + \nu_u^0 \bar{w}^{\dot{\alpha}u}) + \chi^i \bar{\nu}^3 \bar{\nu}_i + \bar{\chi}_i \nu_u^0 \nu^{u,i} \\
&\quad + \overline{A}_{iuv} \nu^{u,i} \bar{\nu}^{0v} + \overline{E}_u \nu^3 \bar{\nu}^{0u} + \overline{C}_3 \nu_u^0 \bar{\nu}^3 + \overline{C}_i^u \nu_u^0 \bar{\nu}^i \qquad (0.1) \\
&\quad + B_i \nu^3 \bar{\nu}^i + C_u^3 \nu^{ui} \bar{\nu}_i + C_u^i \nu^{ui} \bar{\nu}_3 \qquad (0.2) \\
\mathcal{S}_B &= \overline{\chi}_i \chi^i \overline{w}_{\dot{\alpha}}^u w_u^{\dot{\alpha}} + \chi^i \overline{w}_{\dot{\alpha}}^u \overline{w}^{v\dot{\alpha}} \overline{A}_{iuv} + \overline{\chi}_i w_u^{\dot{\alpha}} w_{v\dot{\alpha}} A^{iuv} \\
&\quad + w_u^{\dot{\alpha}} \overline{w}_{\dot{\alpha}}^v [\overline{A}_{ivw} A^{iwu} + \overline{E}_v E^u + C_v^3 \overline{C}_3^u + C_v^i \overline{C}_i^u]
\end{aligned}$$

Fermionic zero-modes, except for Θ^0, lifted by Yukawa-type interactions.

Fermionic integrals bring down 9 scalar fields in the numerator so that $18 = dim \mathfrak{M}^{fermi}_{U(5),k=1} - 2$ fermionic zero modes besides $\Theta^{0\alpha}$ are soaked up.
Bosonic gaussian integrals produce scalar fields in the denominator so that resulting expression for $W(\Phi_f)$ is holomorphic as expected.
Structure of the integrand supports previous derivation of non-perturbative superpotential
Explicit evaluation rather involved ... to be completed

Summary and Outlook

- Unoriented Z_5 quiver GUT with two generations of $\mathbf{10} + \mathbf{5}^*$, one mirror pair $\mathbf{5} + \mathbf{5}^*$ and two charged singlets B_i
- Local description (twisted tadpole cancellation), *e.g.* isolated Z_5 singularity on a Z_5 quotient of the Quintic [Witten]
- For $\xi_{1,5} = 0$ valley of supersymmetric vacua, quantum deformation of the 'moduli' space, yet $B_i = 0$
- For $\xi_1 > 0$, $B_i \neq (0, 0)$, non-perturbative superpotential from 'gauge' instantons $\rightarrow$ dynamical supersymmetry breaking
- Perfect matching between 'microscopic' (UV, weak coupling) description (elementary fields) and 'macroscopic' (IR, strong coupling) description ('baryons' and 'mesons')
- D-instanton calculus to be completed, possibly including fluxes
- Embedding in a global setting (Quintic?), ... mediation to the visible sector

Announcement

- *Strings '09*

  Rome, 22-26 June 2009

  Angelicum - Pontificia Università S. Tommaso
  http://people.roma2.infn.it/ strings2009/