The scale of SUSY breaking in models of inflation driven by the volume modulus

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Outline

- Motivations
- Constraints for the Kähler potential
- Model building
- Conclusions

KKLT moduli stabilization

F-term potential in 4D SUGRA:

$$V = e^K \left(K^{I\bar{J}} D_I W \overline{D_J W} - 3 \left| W \right|^2 \right)$$

Kähler potential for the volume modulus:

$$K = -3 \ln(T + \overline{T})$$

For fixed dilaton and CSM fluxes contribute constant term to the superpotential:

$$W = A$$

Introducing non-perturbative correction (e.g. gaugino condensation) to the superpotential:

$$W = A + Ce^{-cT}$$

volume modulus can be stabilized at AdS SUSY minimum.

We live in dS space $\Rightarrow$ $\overline{D3}$-branes introduced to uplift minimum to dS space:

$$\Delta V = \frac{E}{(T + \overline{T})^2}$$

Hubble scale vs SUSY breaking scale

KKLT stabilization allow for constructing models of inflation within string theory e.g. racetrack inflation with 2 non-perturbative terms in the superpotential:

$$W = A + Ce^{-cT} + De^{-dT}$$

In generic inflationary model based on KKLT moduli stabilization Hubble scale during inflation is related to the gravitino mass (Kallosh, Linde '04):

$$H \lesssim m_{3/2}$$

If we insist on a low energy SUSY breaking, the relation $H \lesssim m_{3/2}$ forces us to construct low scale inflationary models ($H \sim \mathcal{O}(1\mathrm{TeV})$) $\Rightarrow$ very hard to find such models (no moduli inflation model of this type constructed so far)

Large $m_{3/2}$ originates from a deep SUSY AdS minimum before uplifting $\Rightarrow$ for the SUSY Minkowski minimum ($m_{3/2} = 0$) there is no relation between H and $m_{3/2}$

SUSY "almost"-Minkowski minimum + small uplifting $\Rightarrow m_{3/2} \ll H$

Constraints on Kähler potential

Slow-roll inflation requires slow-roll parameter $|\eta| \ll 1$

The maximal value of $\eta_{\rm max}$ is related to the curvature of the Kähler manifold spanned by the scalar fields appearing in the theory. (MB, Olechowski '08; Covi et al. '08)

The necessary condition for $|\eta| \ll 1$ (i.e. $\eta_{\rm max} \gtrsim 0$):

$$R(f^i) < \frac{2}{\widehat{G}^2} < \frac{2}{3}$$

where $G = K + \log |W|^2$ and $\widehat{G}^2 \equiv \sqrt{G^i G_i} = 3 + e^{-G} V$

$R(f^i) \equiv R_{i\bar{j}p\bar{q}} f^i f^{\bar{j}} f^p f^{\bar{q}}$ is the sectional curvature along the direction of the SUSY breaking ($f_i \equiv G_i / \widehat{G}^2$ is the unit vector defining that direction).

Note: $\widehat{G}^2 = 3$ for Minkowski, $\widehat{G}^2 > 3$ for de Sitter.

The above condition can be used to eliminate some models even without specifying the superpotential!

Volume modulus as the inflaton

Kähler potential for the volume modulus:

$$K = -3\ln(T + \overline{T})$$

The curvature scalar takes the form:

$$R_T = \frac{2}{3}$$

The trace of the η-matrix is constant and negative:

$$\eta_t^t + \eta_\tau^\tau = -\frac{4}{3}$$

where $t = \mathrm{Re}T$ and $\tau = \mathrm{Im}T$.

$\eta_{\max} = -2/3 \Rightarrow$ No inflation for any superpotential $\Rightarrow$ corrections to Kähler potential required

Corrections to Kähler potential

The necessary condition for the positivity of the η-matrix trace:

$$R_T < 2/3$$

Sufficient condition:

$$R_T \leqslant 0$$

We consider Kähler potential with leading α'-correction and string loop correction:

$$K = -3\ln(T+\overline{T}) - \frac{\xi_{\alpha'}}{(T+\overline{T})^{3/2}} - \frac{\xi_{\text{loop}}}{(T+\overline{T})^{2}}$$

Curvature scalar for this setup reads:

$$R_T = \frac{2}{3} - \frac{35}{48}\frac{\xi_{\alpha'}}{(T+\overline{T})^{3/2}} - \frac{8}{3}\frac{\xi_{\text{loop}}}{(T+\overline{T})^{2}} + \dots$$

Relatively small corrections could make trace of the η-matrix positive.

No inflation in Kallosh-Linde model

The superpotential in KL model reads:

$$W = A + Ce^{-cT} + De^{-dT}$$

SUSY Minkowski minimum exists for fine-tuned value of A:

$$A = -C\left|\frac{cC}{dD}\right|^{\frac{c}{d-c}} - D\left|\frac{cC}{dD}\right|^{\frac{d}{d-c}}$$

SUSY Minkowski minimum occurs at:

$$T_{\rm mink} = t_{\rm mink} = \frac{1}{c-d}\ln\left|\frac{cC}{dD}\right| , \qquad \tau_{\rm mink} = 0$$

Inflation ending in SUSY Minkowski minimum cannot be realized in KL model even with the corrections to Kähler potential. (MB, Olechowski '08)

Triple gaugino condensation model

The superpotential reads:

$$W = A + Be^{-bT} + Ce^{-cT} + De^{-dT}$$

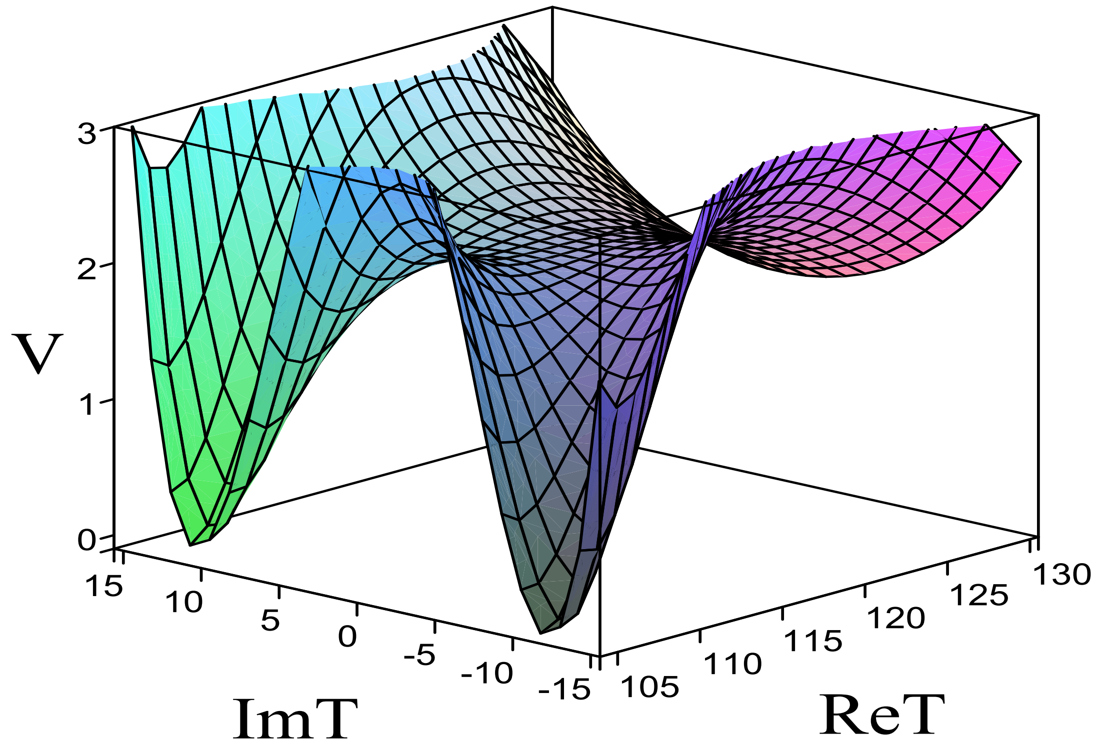
Kähler potential with leading corrections:

$$K = -3\ln(T + \overline{T}) - \frac{\xi_{\alpha'}}{(T + \overline{T})^{3/2}} - \frac{\xi_{\text{loop}}}{(T + \overline{T})^{2}}$$

SUSY Minkowski conditions ($\partial_T W = W = 0$) cannot be solved analytically.
Solution is not unique. There are 2 types of solutions:

- $\tau_{\text{mink}} = 0$ and A real $\rightarrow$ structure of the potential as in KL model $\rightarrow$ no inflation
- $\tau_{\text{mink}} \neq 0$ and A complex $\rightarrow$ inflation can be realized only if B (C or D) complex

Inflationary potential



AdS minimum:	$t_{\rm AdS} = 104.646$,	$\tau_{\rm AdS} = -11.664$
SUSY Minkowski minimum:	$t_{\rm mink} = 104.473$,	$\tau_{\rm mink} = 10.885$
Inflationary saddle point:	$t_{\rm saddle} = 115.475$,	$\tau_{\rm saddle} = -0.183$

Fine-tuning

Triple gaugino condensation model is the first one that accommodates TeV-range gravitino mass and high scale of inflation but requires significant amount of fine-tuning:

Two parameters fine-tuned to (almost) cancel diagonal and off-diagonal entry of the η-matrix $\Rightarrow$ one more fine-tuning than in typical models (e.g. racetrack inflation)

Is this additional tuning necessary in models with light gravitino?

NO, if parameters of W are real and $\tau = 0$ during inflation $\Rightarrow$ off-diagonal entry of the η-matrix vanishes automatically and one tuning is enough

Is it possible to construct such models?

Inflection point inflation

Inflation in the t-direction ($\tau = 0$) can occur in the vicinity of the inflection point.

Naturally realized with positive exponents in gaugino condensation terms.

Positive exponents may occur when gauge kinetic function takes the form:

$$f = w_S S + w_T T\,.$$

Gaugino condensation generates:

$$W_{\rm np} = B e^{-\frac{2\pi}{N}(w_S S + w_T T)}\,.$$

When $w_T < 0$ and dilaton S is stabilized at higher scales, positive exponents appear in effective theory for the volume modulus:

$$W_{\rm np}^{\rm eff} = B^{\rm eff} e^{bT}\,.$$

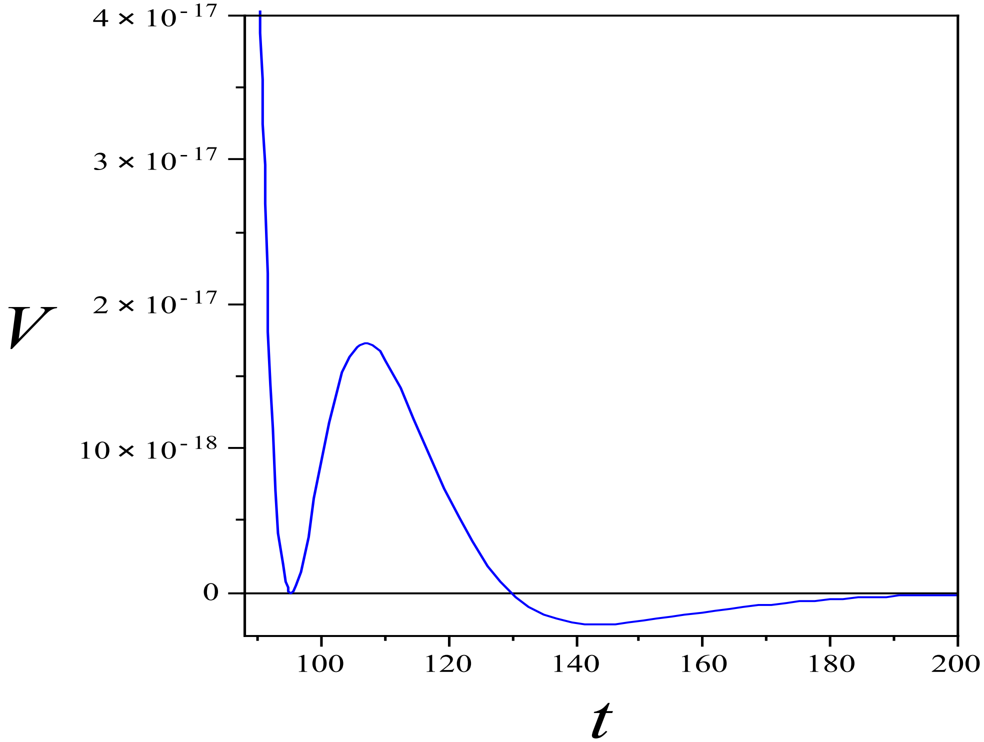
Gauge kinetic functions of this type realized in string theory
(Marchesano, Shiu '04; Cascales, Uranga '03; Lukas, Ovrut, Waldram '97)

Model building with positive exponents

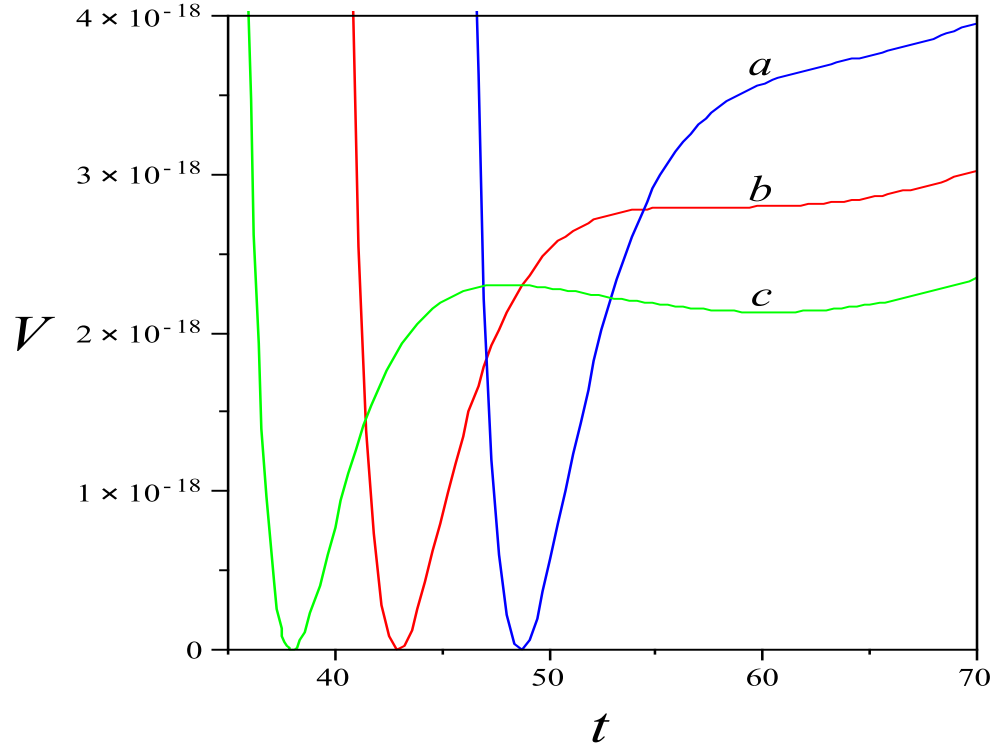
We consider superpotential with 2 gaugino condensates:

$$W = A + Ce^{cT} + De^{dT} .$$

$c < 0$, $d < 0$ (KL model)
SUSY AdS minimum
no inflation with light gravitino



$c < 0$, $d > 0$ and $c > 0$, $d > 0$
nonSUSY AdS/dS minimum
inflection point inflation



What is the price for the working model?

Fine-tuning of one parameter (e.g. C) at the level of 10^{-5} (similar to racetrack inflation).

Stabilization of the τ-direction through string corrections to tree-level Kähler potential.

Fine-tuning of the initial conditions for t at the level of one percent (comparable to other models of small-field inflation).

TeV-range gravitino mass requires:

Fine-tuning of A at the level of 10^{-5}.

Threshold corrections to gauge kinetic function

We consider superpotential with 1 gaugino condensate and threshold corrections:

$$W = A + (C_0 + C_1 T)e^{cT} .$$

SUSY Minkowski minimum exists for fine-tuned value of A:

$$A = \frac{C_1}{c}\exp\left(-\frac{cC_0}{C_1} - 1\right) .$$

SUSY Minkowski minimum occurs at:

$$T_{\mathrm{Mink}} = -\frac{1}{c} - \frac{C_0}{C_1} .$$

For positive c inflection point inflation with light gravitino can be realized

The same conditions for successful inflation as in double gaugino condensation model

Very modest model $\rightarrow$ only 4 parameters in W

$\rightarrow$ 1 parameter in K (to stabilize the τ direction)

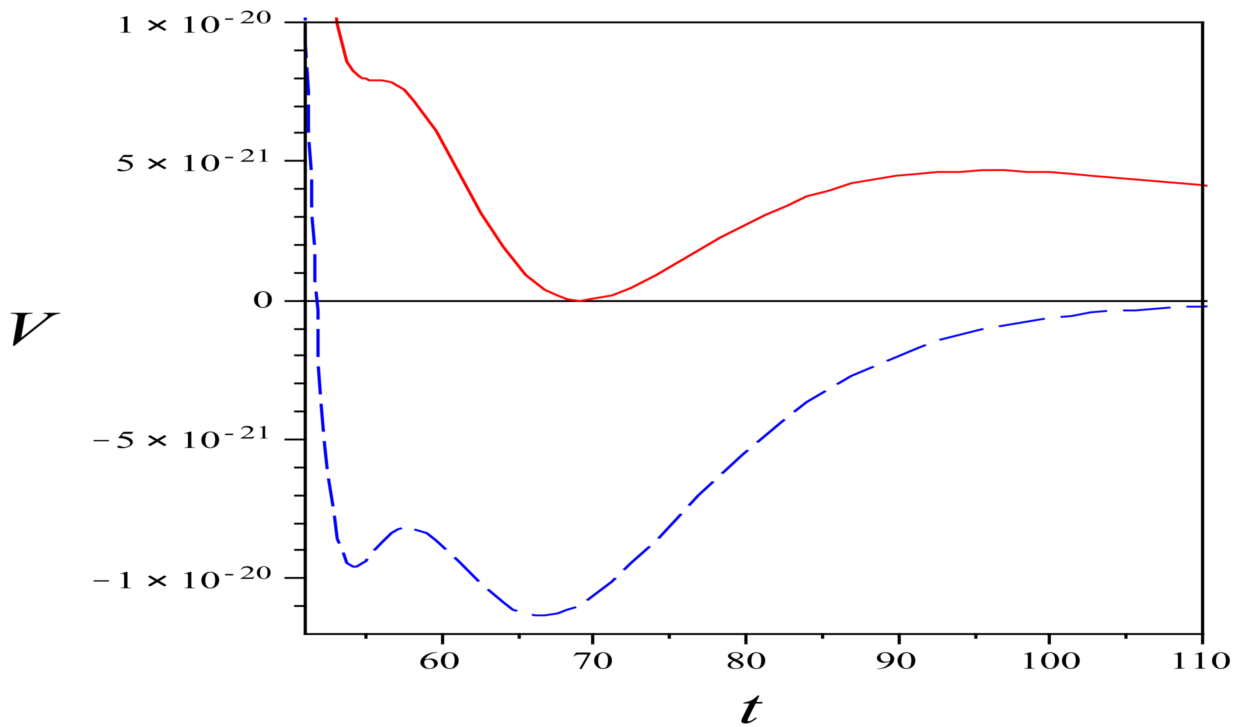
Negative exponents and inflation with heavy gravitino

Without positive exponents inflation possible only with uplifting (heavy gravitino)

$$\Delta V = \frac{E}{t^2} \; .$$

Examples: $\longrightarrow$ KL model (Linde, Westphal '07)

$\longrightarrow$ Single gaugino condensation with threshold corrections



Fine-tuning and the overshooting problem

Fine-tuning of the potential and the initial conditions is inevitably related to the height of the barrier that protects the inflaton from overshooting Minkowski vacuum.

Uplifting is decreasing function of $t \Rightarrow$ maximum (before uplifting) necessarily much below 0 if the barrier is to be high (after uplifting)

Fine-tuning of the parameters in models with negative exponents (heavy gravitino) at least $10^{-8} \Rightarrow$ substantially bigger than in models with positive exponents (light gravitino)

Supersymmetric uplifting from matter field

In models with negative exponents non-supersymmetric uplifting can be substituted by SUSY breaking matter field sector (MB, Olechowski, in preparation):

$$W_{\text{matter}} = c_0 + \mu^2 \Phi \qquad K_{\text{matter}} = |\Phi|^2 - \frac{|\Phi|^4}{\Lambda^2}$$

- $\Lambda \to \infty$ - Polonyi model

  $m_\Phi \sim m_T \Rightarrow$ the inflaton is a mixture of T and Φ

  Advantage: fine-tuning no longer related to the height of the barrier

- $\Lambda \ll 1$ - O'KKLT model

  $m_\Phi \sim \Lambda^{-1} \Rightarrow m_\Phi \gg m_T$ - matter field decoupled from the inflationary dynamics but provides spontaneous SUSY breaking and uplifting

Conclusions

- For the volume modulus, parameter η necessarily smaller than $-2/3$ and inflation with TeV-range gravitino mass cannot be realized unless corrections to the leading Kähler potential are included.

- Even for corrected Kähler potential inflation cannot be realized in KL model with only two non-perturbative terms in the superpotential.

- Adding third non-perturbative term to the superpotential makes inflation possible but significant amount of fine-tuning is necessary.

- Positive exponents in non-perturbative terms help in realizing inflection point inflation with light gravitino. Double gaugino condensation or single one with threshold corrections are enough to realize successful models.

- Inflection point models with all exponents negative can be realized only with uplifting (heavy gravitino) and suffer from overshooting problem.

- To overcome overshooting problem in models with all exponents negative parameters has to be much more fine-tuned than in models with positive exponents where overshooting problem is absent.

- Fine-tuning in models with all exponents negative can be relaxed if SUSY is broken by the matter field with a mass comparable to the volume modulus (e.g. Polonyi model). In such case fine-tuning is not related to the height of the barrier and is comparable to the fine-tuning in models with positive exponents.